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Features of the Application of Compound Codes in Telecommunications

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ABSTRACT In telecommunications systems and networks designed for information transmission, independent and packet errors may occur simultaneously. To correct this type of error, it is advisable to use a code that can correct both packet and independent errors. Compound codes are one type of cyclic code. They are a type of systematic code. They are uniform, divisible, and block-based. They have an exceptional property: if a code combination belongs to a code, then a new combination obtained by cyclic permutation of bits also belongs to that code. These codes have a simple hardware implementation of encoding/decoding schemes and high efficiency in detecting and correcting errors. This, in turn, has ensured their widespread use. Irreducible polynomials are used to construct cyclic codes. To obtain a compound code, it is necessary to multiply the generating polynomial of the Fire code by the generating polynomial of the Bose-Chaudhuri-Hocquenghem (BCH) code. In reference literature, compound codes are usually denoted as FxBCH. A comparison of codes that correct only error packets and compound codes showed that compound codes have a greater number of check digits. Most of the obtained optimal compound codes can be used in practice in coding systems. The article studies compound codes that combine several coding methods to improve the reliability of data transmission in telecommunications systems. Their structure, advantages, and disadvantages compared to traditional codes, such as Hamming and Reed-Solomon codes, are considered. The effectiveness of compound codes in noisy channels is analyzed, including numerical calculations of error probabilities and performance comparisons. The results are presented in tables and graphs showing the dependence of effectiveness on channel parameters. The work aims to determine the optimal conditions for the use of compound codes in modern telecommunications systems, particularly in the context of 5G and promising 6G technologies.

KEYWORDS compound codes, interference, corrective property, optimality, redundancy.

I. INTRODUCTION

Modern telecommunications systems, such as 5G networks, high-definition television (HDTV), satellite communications, and Internet of Things (IoT) systems, require high reliability of data transmission in channels with various types of interference. Compound codes, which are a combination of several codes (e.g., block and convolutional codes), offer an effective solution for ensuring error resilience. They allow the strengths of different coding methods to be combined, making them attractive for use in high-noise environments.

The purpose of this article is to analyze the structure and efficiency of compound codes, compare them with traditional coding methods, and evaluate their suitability for modern telecommunications systems. The paper uses mathematical models, numerical calculations, tables, and graphs to illustrate the results.

II. ANALYSIS OF COMPOUND CODE FEATURES

A compound code is a combination of two or more codes that are applied sequentially or in parallel to increase fault tolerance. For example, an external code (usually block-based, such as Reed-Solomon code) is combined with an internal code (convolutional or turbo code). This structure allows you to:
- correct both single and batch errors;
- increase channel throughput through adaptive coding;
- provide flexibility in configuring parameters depending on channel conditions.

Although compound codes inherit some structural features of cyclic codes – such as block organization and cyclic invariance – their key advantage lays in the integration of multiple error correction mechanisms. This hybrid nature allows compound codes to correct both burst and isolated errors more effectively than traditional cyclic codes, which is critical in dynamic environments such as 5G and satellite communications. These codes have a simple hardware implementation of encoding/decoding schemes and high efficiency in detecting and correcting errors. This, in turn, has ensured their widespread use [1]. Code combinations of cyclic codes are conveniently presented as a polynomial of degree n-1, where n is the number of digits of the CC (code combination):

$$F(x) = a_{n-1} \cdot x^{n-1} + a_{n-2} \cdot x^{n-2} + \cdots + a_1 \cdot x^1 + a_0, \quad (1)$$

where x is the base of the number system, a_i are the digits of the given number system, and in the binary number system under consideration, a_i can take the values “0” and “1.”

Irreducible polynomials are used to construct cyclic codes.

The construction of compound codes in this study involves multiplying the generating polynomials of a Fire code and a Bose-Chaudhuri-Hocquenghem (BCH) code. This approach results in a compound generator polynomial with improved minimum distance, enhancing the code’s ability to detect and correct both isolated and clustered errors – a key requirement in next-generation wireless and satellite networks.

To obtain a compound code, it is necessary to multiply the generating polynomial of the Fire code by the generating polynomial of the BCH code. In reference literature, compound codes are usually denoted as FxBCH [2].

Mathematically, if the outer code has parameters $(n_1,\ k_1, d_1)$, where n_1 is the code word length, k_1 is the number of information symbols, d_1 s the minimum code distance, and the internal code has parameters $(n_2,\ k_2, d_2)$, then the resulting compound code will have parameters:

$$(n,k,d) = (n_1 \cdot n_2,\ k_1 \cdot\ k_2, d_1 \cdot d_2). \qquad (2)$$

This means that compound codes inherit the ability to correct errors from both components, making them particularly effective in complex channels. The corrective property of compound codes can reach high values, such as: code distance d_{min} = 5 – 21 and above, packet error length b = 3 – 20 and above.

Code redundancy is defined as the ratio of the number of redundant symbols to the total length of the code word. Mathematically, this is expressed by the formula:

$$r = \frac{n-k}{n}, \qquad (3)$$

where: n is the total length of the code word, k is the number of information symbols.

According to this formula, redundancy r is calculated. The results of the calculations are presented below in Table 1, where redundancy r is rounded to four decimal places.

Redundancy varies from 0.1173 (lowest for n = 341, k = 301) to 0.2466 (highest for n = 73, k = 55). Codes with a longer code word length (n = 1023) have lower redundancy compared to shorter codes, which may indicate efficient use of symbols. With an increase in d_{min} and b (error correction capability) redundancy usually increases, which is a compromise between reliability and channel efficiency. Accordingly, Fig. 1 shows the dependence of the minimum code distance d_{min} on redundancy r.

TABLE 1. Corrective properties of compound codes.

n	*k*	d_{min}	*b*	*R*
73	55	5	6	0.2466
127	106	7	5	0.1654
341	301	9	16	0.1173
381*	325	11	25	0.1470
341	291	11	23	0.1466
381	311	13	30	0.1613
341	286	13	24	0.2205
381	297	15	40	0.2205
1023*	902	17	56	0.1183
1023	888	19	64	0.1320
1023*	872	21	72	0.1476

*Shortened cyclic codes are marked, the rest are cyclic codes.

Comparative analysis shows that compound codes generally include more check symbols than codes optimized solely for burst error correction. However, this redundancy is offset by the enhanced ability to simultaneously correct both independent and packet errors. This dual correction capability makes compound codes optimal for communication scenarios with mixed noise characteristics, such as those found in 5G base stations or mobile backhaul links.

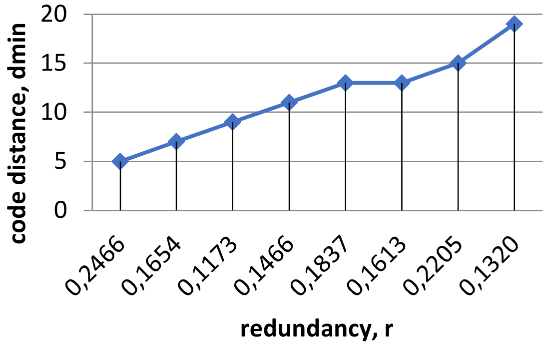

FIG. 1. Dependence of the minimum distance d_{min} on redundancy r.

Most of the optimal compound codes obtained can be used in practice in coding systems.

This is because the complexity of the decoding device is of the same order as that of BCH code decoders. Below, in Table 2, are the optimal compound codes that correct independent errors and for which the transmission rate is not less than the transmission rate of any other codes with the same CC length and correction capacity d_{min}.

TABLE 2 Optimal compound codes that correct independent errors.

n	*k*	d_{min}	*b*
73	46	9	-
127	92	11	-
151	91	11	28
151	106	7	20
381	121	5	12
381	297	15	40
381	311	13	30
381	325	11	25

Optimal compound codes are characterized by high coding and decoding efficiency and the ability to ensure reliable data transmission in various interference conditions.

Compound codes include turbo codes, LDPC codes, convolutional codes, block codes, and Reed-Solomon codes [3]. Compound codes can include combinations of different types of encodings, such as block encoding, convolutional encoding, and others. By combining these methods, it is possible to achieve greater efficiency and fault tolerance, which is especially important in modern telecommunications systems where signal quality can be low.

III. ANALYSIS OF THE EFFICIENCY OF COMPOUND CODES

For evaluating the efficiency of compound codes [4], let us consider a binary symmetric channel (BSC) with error probability p. The decoding error probability for a code with minimum code distance d_{min} can be estimated using the following formula:

$$P_e \approx \sum_{i=t+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}, \qquad (4)$$

where: $t = (d-1)/2$ is the number of errors that the code can correct.

Let's look at an example of a compound code consisting of a Reed-Solomon code (255, 223, 33) as the outer code and a wrapper code (2, 1, 7) as the inner code. For such a code, the outer code corrects up to $t_1 = \frac{(33-1)}{2} = 16$ errors. The inner code with a free distance $d_f = 7$ corrects up to $t_1 = \frac{(7-1)}{2}$ =3 errors.

Let us calculate the probability of decoding error for different values of p in Table 3.

TABLE 3. Decoding error probability for different codes.

p	P_e (compound code)	P_e (Rida-Solomon)	P_e (convolutional codes)
0.01	$1.2 \cdot 10^{-5}$	$3.5 \cdot 10^{-4}$	$2.1 \cdot 10^{-3}$
0.05	$4.8 \cdot 10^{-3}$	$1.9 \cdot 10^{-2}$	$3.7 \cdot 10^{-2}$
0.1	$2.3 \cdot 10^{-2}$	$8.4 \cdot 10^{-2}$	$1.2 \cdot 10^{-1}$

As can be seen from Table 3, the compound code significantly reduces the probability of errors compared to individual codes, especially at high p values.

For clarity, let us consider the dependence of the bit error rate (BER) on the signal-to-noise ratio (SNR) for compound code, Reed-Solomon code, and convolutional code. The simulation results are shown in Fig. 2.

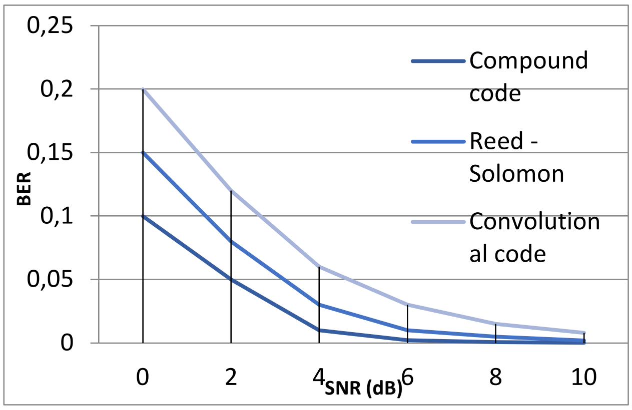

FIG. 2. BER dependence on SNR for different codes.

The graph shows that the compound code achieves a lower BER at lower SNR values, confirming its effectiveness in channels with low signal-to-noise ratios.

Let's take a closer look at their advantages and disadvantages.

Compound codes effectively handle various types of errors, both single and packet errors [5]. For example, an external code (such as Reed-Solomon or BCH) corrects error packets that occur due to short-term signal interruptions (for example, in satellite channels due to rain), while an internal code (convolutional or LDPC) effectively corrects distributed errors caused by noise. This ensures reliable data transmission in challenging environments such as satellite communications (DVB-S2) or 5G networks.

Compound codes allow encoding parameters such as code word length (n), number of information symbols (k) and minimum code distance (d_{min}), to be adapted to specific channel conditions. For example, in systems with dynamic conditions (mobile networks), the coding rate ($R = k/n$) can be changed to balance between error resilience and throughput. This is particularly important for 5G and 6G technologies, where channel characteristics can change rapidly due to user mobility or interference.

By combining codes with different characteristics, composite codes allow you to achieve the optimal balance between redundancy and data transfer speed. For example, in channels with low noise levels, you can use a code with lower redundancy ($r = (n-k)/n$), which increases effective throughput. In DVB-S2, adaptive coding and modulation allows the coding rate to be dynamically adjusted (e.g., from 1/4 to 9/10), providing high throughput (up to 50 Mbit/s in a 36 MHz channel).

Compound codes are widely used in telecommunications standards such as DVB-S2, LTE, 5G NR, and WiMAX [6, 7]. For example, in DVB-S2, the combination of BCH and LDPC codes provides a frame error rate below 10^{-7} even at low SNR values (3 dB – 5 dB), which is ideal for HDTV and 4K video transmission.

In systems with variable channel conditions (e.g., wireless networks), compound codes allow the coding structure to be dynamically changed depending on the interference level. For example, in 5G NR, adaptive coding allows switching between different code combinations (LDPC + polar codes) to ensure optimal performance in real time [8].

Compound codes demonstrate high efficiency in channels with a combination of Gaussian noise, impulse noise, and fading. For example, in an AWGN (additive white Gaussian noise) channel, the BCH+LDPC compound code provides a bit error rate (BER) of 10^{-7} at SNR = 4 dB, which is significantly better than for individual codes.

Let's list the main disadvantages of compound codes. Compound codes, especially those that use iterative decoding methods (e.g., turbo codes or LDPC in combination with BCH), require significant computing resources. For example, decoding an LDPC code in DVB-S2 requires thousands of iterations based on the belief propagation algorithm, which can be problematic for devices with limited computing capabilities, such as IoT devices or budget mobile terminals [9].

The sequential application of external and internal codes (e.g., BCH → LDPC) results in additional delays in the encoding and decoding process. In DVB-S2, decoding delays can reach several milliseconds, which can be critical for real-time applications such as VoIP, streaming video, or interactive games, where delay requirements are less than 1 ms.

The efficiency of compound codes depends on the correct selection of external and internal code parameters (coding rate, code distance, code word length). Incorrect matching can lead to excessive redundancy or reduced error resilience. For example, if the external BCH code in DVB-S2 has too much redundancy, it can reduce throughput without significantly improving reliability.

Compound codes usually have higher redundancy than individual codes, which reduces the effective data transfer rate. For example, for a code with parameters (n = 73, k = 55, d_{min} = 5), the redundancy is r = (73 - 75) ≈ 0.2466, which means that almost 25% of the code word is redundant bits. In low-noise channels, this may be an unjustified compromise.

The implementation of compound codes in hardware (e.g., in FPGA or ASIC) is more complex than traditional codes due to the need to integrate several encoding and decoding algorithms. For example, DVB-S2 requires the implementation of both a BCH decoder (based on the Berges-Messier algorithm) and an LDPC decoder (based on iterative methods), which increases the cost of equipment development and production [10].

The high computational complexity of decoding compound codes leads to increased energy consumption, which can be problematic for battery-powered devices such as smartphones or IoT sensors. For example, iterative decoding of LDPC codes in 5G devices can significantly reduce battery life.

For clarity, let us present the main advantages and disadvantages of compound codes in Table 4.

TABLE 4. The main advantages and disadvantages of compound codes.

Aspect	Advantages	Disadvantages
Fault tolerance	Correction of single and packet errors, high reliability in noisy channels	Requires precise coordination of parameters for maximum efficiency
Throughput capacity	Optimization through adaptive coding	Increased redundancy reduces effective transmission speed
Flexibility	Adaptation to different channel conditions	The complexity of selecting optimal parameters
Computational complexity	-	High demands on computing resources and energy consumption
Delay	-	Increased latency due to cascaded encoding structure
Implementation	Compatibility with modern standards (DVB-S2, 5G)	Complexity of hardware implementation (FPGA, ASIC)

V. CONCLUSION

The paper discusses the features of compound codes and their role in modern telecommunication systems. Compound codes belong to cyclic codes. The corrective property of compound codes can reach high values with the possibility of correcting packet and independent errors. When using optimal codes, it is possible to achieve high coding and decoding efficiency and ensure reliable data transmission in various interference conditions.

Compound codes can reduce the number of data retransmissions, which in turn saves bandwidth and reduces information processing costs. Due to their flexibility, compound codes can be adapted to communication channels, making them versatile for various applications.

But there are also some downsides. First, they're tricky to implement in terms of system design and maintenance. Second, they use more computing resources, which is something to keep in mind when designing systems with limited resources.

Research has shown that compound codes are a powerful tool for improving the reliability of telecommunications systems, especially in high-noise environments such as satellite channels or 5G networks. Their advantages, such as high fault tolerance, flexibility, and compatibility with modern standards, make them indispensable in many applications. Analytical calculations and graphical analysis confirm their advantage over traditional codes in high-noise channels. However, their use is limited by computational complexity and delays, which requires further optimization of decoding algorithms. Future research plans to study adaptive compound codes for 6G networks, as well as their integration with artificial intelligence technologies for dynamic coding parameter adjustment.

AUTHOR CONTRIBUTIONS

O.Y. – writing (original draft preparation), conceptualization, methodology, investigation; O.S. – methodology, investigation, writing (review and editing).

COMPETING INTERESTS

The authors declare that they have no conflict of interest

REFERENCES

[1] M. Burachok, M. Klymash, and B. V. Koval, *Telecommunication Systems for Information Transmission. Encoding Methods*. Lviv: Lviv Polytechnic Publishing House, 2015.

[2] I. P. Panfilov, V. Yu. Dyda, and A. V. Kapatsin, *Theory of Electrical Communication*. Kyiv: Tekhnika, 1998.

[3] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near Shannon limit error-correcting coding and decoding: Turbo codes,” *IEEE Trans. Inf. Theory*, vol. 44, no. 10, pp. 1064–1070, 1996.

[4] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes*. North-Holland, 1977.

[5] C. E. Shannon, “A mathematical theory of communication,” *Bell Syst. Tech. J.*, vol. 27, pp. 379–423, 623–656, 1948.

[6] J. G. Proakis, *Digital Communications*, 5th ed. New York: McGraw-Hill, 2008.

[7] T. K. Moon, *Error Correction Coding: Mathematical Methods and Algorithms*. Hoboken, NJ: Wiley-Interscience, 2005.

[8] S. Lin and D. J. Costello, *Error Control Coding: Fundamentals and Applications*, 2nd ed. Pearson, 2004.

[9] L. Hanzo, T. Liew, and B. Yeap, *Turbo Coding, Turbo Equalisation and Space-Time Coding*. Wiley, 2002.

[10] ETSI EN 302 307, “Digital Video Broadcasting (DVB); Second generation framing structure, channel coding and modulation systems for Broadcasting, Interactive Services, News Gathering and other broadband satellite applications (DVB-S2),” ETSI, 2009.

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Особливості застосування компаундних кодів в телекомунікаціях

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АНОТАЦІЯ У телекомунікаційних системах і мережах, призначених для передачі інформації, можливе одночасне виникнення незалежних і пакетних помилок. Для виправлення цього типу помилок доцільно використовувати код, який може виправляти як і пакетні, так і незалежні помилки. До таких циклічних кодів відносяться компаундні коди. Це різновид систематичних кодів. Вони є рівномірними, подільними, блочними. Мають виняткову властивість - якщо кодова комбінація належить коду, то нова комбінація, яка отримана циклічною перестановкою бітів, також належить до цього коду. Ці коди мають просту апаратну реалізацію схем кодування/декодування і високу ефективність виявлення і виправлення помилок. Що в свою чергу забезпечило їх широке застосування. Для побудови циклічних кодів використовують неприводимі многочлени (поліноми). Для отримання компаундного коду необхідно перемножити породжуючий поліном коду Файра на породжуючий поліном коду Боуза-Чоудхурі-Хоквінгема (БЧХ). У довідниковій літературі компаундні коди переважно позначаються як ФхБЧХУ. Порівнюючи коди, які виправляють лише пакети помилок і компаундні коди, показало, що компаундні коди мають більшу кількість перевірочних розрядів. Більшість з отриманих оптимальних компаундних кодів можливо використовувати на практиці в системах кодування. У статті проведено дослідження компаундних кодів, які поєднують кілька методів кодування для підвищення надійності передачі даних у телекомунікаційних системах. Розглянуто їхню структуру, переваги та недоліки порівняно з традиційними кодами, такими як коди Хеммінга та Ріда-Соломона. Проведено аналіз ефективності компаундних кодів у каналах із завадами, включаючи числові розрахунки ймовірності помилок і порівняння продуктивності. Результати представлено у вигляді таблиць і графіків, що демонструють залежність ефективності від параметрів каналу. Робота спрямована на визначення оптимальних умов застосування компаундних кодів у сучасних телекомунікаційних системах, зокрема в контексті 5G і перспективних технологій 6G.

КЛЮЧОВІ СЛОВА компаундні коди, завади, коригуюча властивість, оптимальність, надлишковість.