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# **Analysis of Results of Scaling Digital Images by Interpolation Algorithms**

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**ABSTRACT** Scaling of digital bitmap images is often used in modern computer and telecommunications systems. Among image scaling algorithms, the most common are interpolation algorithms, namely nearest neighbor, Lanczos, bilinear and bicubic interpolations. However, in the process of scaling images by interpolation algorithms, characteristic distortions appear on them. Therefore, in this work, software implementation and research of image interpolation algorithms was performed in order to identify their advantages and disadvantages, areas of use and ways of improvement. In order to study the interpolation algorithms, a scaled fRGBs image was calculated based on the initial image fRGB, and then a scaled image fRGBs2 with the dimensions of the original image was calculated based on fRGBs. The scaling accuracy is evaluated using the Root Mean Square Error between the pixel values of the original and scaled images. The software for scaling images is developed in the Python language. Calculation of scaled images is performed by the cv2.resize() function of the OpenCV library. Using the developed program, scaling of a series of 100 images was carried out, the accuracy and speed of digital image scaling by interpolation algorithms were investigated. For each interpolation method, the average scaling error AR and the limits of its confidence interval ARmin and ARmax with a specified reliability γ are calculated. The average scaling time AT of a series of 100 images by different algorithms was determined. The research results showed that in most cases the smallest scaling error is provided by the bicubic interpolation algorithm, which is slightly inferior in speed to the nearest neighbor and bilinear interpolation algorithms. Recommendations for the application of interpolation algorithms have been developed. It is shown that the use of convolutional neural networks is promising for the highest quality image scaling.

**KEYWORDS** scaling of digital images, nearest neighbor algorithm, bilinear Interpolation, bicubic interpolation, Lanczos algorithm.

## **I. INTRODUCTION**

caling of digital raster images is an important task in modern computer and telecommunication systems [1, 2]. Reducing or increasing the scale of **S** caling of digital raster images is an important task in modern computer and telecommunication systems [1, 2]. Reducing or increasing their resolution. Scaling down of images is used, for example, to reduce the load on the channels of the telecommunications system and to reduce the amount of memory for saving images. Scaling up of images is used, for example, to restore the original dimensions of images and to improve their further computer processing [3]. From the point of view of digital signal processing, the scaling of raster images is a two-dimensional example of the conversion of the sampling rate for a discrete signal [4].

Among digital image scaling algorithms, the most common are interpolation algorithms, which are characterized by simplicity of software or hardware implementation, high speed, and relatively high visual quality of scaled images [5]. The following image interpolation algorithms are common: nearest neighbor, Lanczos, bilinear and bicubic interpolations [2, 6]. Other methods of image scaling, in particular, using the Fourier transform [1], multiscale processing [7] and artificial neural networks [8, 9], are characterized by more complex implementation and lower speed.

by interpolation algorithms, characteristic distortions (artifacts) appear on them, contours are blurred, image detail is lost. It is important that different interpolation algorithms distort the image in different ways and have different speeds. Therefore, the goal of the work is relevant, which consists in the software implementation and research of image interpolation algorithms in order to identify their advantages and disadvantages, areas of use and ways of improvement.

## **II. THEORETICAL BASICS OF IMAGE INTERPOLATION ALGORITHMS**

Interpolation algorithms are used for scaling (resizing) images. Digital color images are processed as *fRGB* (*i*, *k*, *c*) arrays, where  $i = 0,..., M-1$ ;  $k = 0,..., N-1$ ;  $c = 0,..., Qc-1$ ; *M* is the number of image pixels by height, *N* is the number of pixels by width;  $Q_C = 3$  is the number of color channels (Red, Green, Blue) [1, 2]. The values of all color channels are normalized in the range from 0 to 1.

Let's consider the principles of the following interpolation algorithms: nearest neighbor, bilinear and bicubic interpolations, Lanczos.

The nearest neighbor algorithm is the simplest. The principle of its operation boils down to the selection of one of the pixel values that are closest to the new pixel position after resizing the image.

The problem is that in the process of scaling images

The bilinear image interpolation algorithm is that the

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new pixel value is calculated using a linear combination of the values of the 4 pixels that are closest to the new pixel position. In the case of image scaling, linear interpolation is performed first in one direction (height) and then in another (width).

The bicubic image interpolation algorithm is that the new pixel value is calculated using the cubic interpolation of the pixel values in two coordinates (height and width). In the case of cubic interpolation, the value of the function (pixel brightness) at the desired point is calculated from its value at 16 neighboring points.

The Lanczos algorithm uses the Lanczos window function (kernel) to calculate a new pixel value based on the values of neighboring pixels when the image is resized. In the case of interpolation of one-dimensional signals  $s(x)$ , the function  $sinc(x/a)$  is used as the Lanczos kernel, where  $-a \le x \le a$ . The two-dimensional Lanczos kernel is used in image processing. The parameter *a* is a positive integer (usually  $a = 3$ ) that defines the size of the kernel.

In order to study interpolation algorithms based on the initial image *fRGB*, the calculation of the scaled image *fRGBs* with the size of  $M_s \times N_s$  pixels (with the scaling factor  $S_c$ , for example,  $S_c = 0.5$ ) was performed, and then, based on *fRGBs*, the calculation of the scaled image *fRGBs*<sup>2</sup> with the dimensions of the initial image of  $M \times N$  pixels is performed.

Ideally, the original *fRGB* and the scaled *fRGBs*<sup>2</sup> images should match. However, due to scaling, there are differences between the images. Scaling error are calculated from the difference between the *fRGB* and *fRGBs*<sup>2</sup> images as the Root Mean Square Error (RMSE) using the equation:

$$
R_{MSE} = \sqrt{\frac{1}{MN} \sum_{i=0}^{M-1} \sum_{k=0}^{N-1} [f_{RGB}(i,k) - f_{RGBS2}(i,k)]^2}
$$
 (1)

The average value of RMSE for a series of *QIm*  $(Q<sub>Im</sub> \ge 10)$  images is denoted by  $A<sub>R</sub>$ , and the standard deviation of RMSE values for a series of *QIm* images is denoted by σ*R*. Testing of the hypothesis of a normal distribution of RMSE values is performed using the Pearson's  $\chi^2$  test [10]. The hypothesis of a normal distribution is confirmed if  $\chi^2$ <sub>R</sub> <  $\chi^2$ <sub>T</sub>( $\alpha$ , k), where  $\chi^2$ <sub>R</sub> is the observed value of the Pearson test,  $\chi^2$ <sup>T</sup> is the critical point of the right-hand region for the  $\chi^2$  distribution, α is the significance level,  $k = (Q_h - 3)$  is quantity of the degrees of freedom,  $Q_h$  is the quantity of histogram intervals. The selection of the optimal  $Q_h$  is performed according to the Sturges' rule [11]

$$
Q_h = \log_2(Q_{\text{Im}}) + 1,\tag{2}
$$

where  $Q_{Im}$  is the number of objects (images).

If the hypothesis of a normal RMSE distribution is confirmed, it is possible to construct a confidence interval  $A_{Rmin} < A_R < A_{Rmax}$  for the average scaling error  $A_R$  with a specified reliability γ [10]. The lower limit *ARmin* of the confidence interval is calculated by the formula

$$
A_{R\min} = A_R - \frac{x_L \cdot \sigma_R}{\sqrt{Q_{\text{Im}}}}\,,\tag{3}
$$

where  $x_L$  is the value of the Laplace function for which

 $\Phi(x_L) = \gamma/2$ . Similarly, the upper limit  $A_{Rmax}$  of the confidence interval is calculated using the formula

$$
A_{R\max} = A_R + \frac{x_L \cdot \sigma_R}{\sqrt{Q_{\text{Im}}}}\,. \tag{4}
$$

Thus, for each interpolation method, it is possible to scale a series of images and calculate not only the average scaling error  $A_R$ , but also the limits  $A_{Rmin}$  and  $A_{Rmax}$  of its confidence interval.

## **III. SOFTWARE IMPLEMENTATION OF IMAGE INTERPOLATION ALGORITHMS**

The image scaling software was developed in Python using OpenCV (cv2), Numpy, Matplotlib, etc. libraries. Calculation of scaled images is performed by the cv2.resize() function of the OpenCV library using the following interpolation methods:

- cv2.INTER\_NEAREST is a method based on the nearest neighbor algorithm, which is characterized by simplicity and high speed, but can lead to loss of details in the image.
- cv2.INTER\_LINEAR is a method based on the linear interpolation algorithm. The method usually gives better results (compared to INTER\_NEAREST), but it smoothes the image to some extent.
- cv2.INTER\_CUBIC is a method based on the cubic interpolation algorithm. The method usually gives better results (compared to INTER\_LINEAR), but is slower.
- cv2.INTER\_LANCZOS4 is a method based on the Lanczos algorithm, which has a relatively low speed (compared to the cv2.INTER\_CUBIC method). Using this method allows to get scaled images with high accuracy, but the appearance of unwanted artifacts such as ringing is possible.

## **IV. ANALYSIS OF RESULTS FOR IMAGE INTERPOLATION ALGORITHMS**

Using the developed software, a series of *QIm* images was read from graphic files and scaled (Fig. 1).

On the basis of each initial *fRGB* image (Fig. 2), a scaled image  $f_{RGBs}$  with the size of  $M_s \times N_s$  pixels with a scaling factor of  $S_c = 0.5$  ( $M_s = M \cdot S_c$ ,  $N_s = N \cdot S_c$ ) was calculated using the bicubic interpolation algorithm (Fig. 3a). Next, based on *fRGBs*, a scaled image *fRGBs*<sup>2</sup> with the dimensions of the initial image  $M \times N$  pixels was calculated (Fig. 3b). Similarly, image scaling was performed using the bilinear interpolation, Lanczos and nearest neighbor algorithms.



**FIG. 1.** Image base BSDS300 [12, 13], 8 images from 100 are shown.

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**FIG. 2.** Initial image *fRGB* #01.



**FIG. 3.** Image *fRGBs* after scaling down (a) and image *fRGBs*<sup>2</sup> after scaling up (b) using the 'CUBIC' method.

Distortions that occur when scaling images are especially noticeable on their profiles (Fig. 4).

For each scaled image *fRGBs*2, a certain value of scaling error RMSE (1) is obtained (Fig. 5).

For most images, the scaling error RMSE by the 'LINEAR' method is greater than by the 'CUBIC' method (Fig. 6); the scaling error by the 'LANCZOS4' method is greater than by the 'LINEAR' method (Fig. 7); the scaling error by the 'NEAREST' method significantly exceeds the errors of other methods (Table I).

The test of the hypothesis of a normal distribution of RMSE values was performed according to the Pearson's  $\chi^2$  test for the significance level  $\alpha = 0.05$ . The optimal quantity of intervals of the histogram  $Q_h = 8$  is calculated



**FIG.** 4. Fragments of the RGB profiles  $z(k)$  of the initial image *fRGB* (a) and the 'CUBIC' method scaled image *fRGBs*<sup>2</sup> (b) for the red, green and blue components; *i<sup>p</sup>* is the pixel row number.

RMSE; QIm=100; met=CUBIC



**FIG.** 5. Values of scaling error RMSE for  $Q_{lm} = 100$  images (Fig. 1), scaled by the 'CUBIC' method;  $n_i$  is image numbers.

according to formula (2), so the quantity of the degrees of freedom is  $k = 5$ . According to the tabular data, the critical point is  $\chi^2(\text{I}(0.05, 5)) = 11.07$ . For the 'CUBIC', 'LINEAR', 'LANCZOS4', 'NEAREST' methods, the observed values of the Pearson's  $\chi^2$ <sub>R</sub> test were obtained,



**FIG. 6.** З Dependence of the scaling error RMSE by the 'LINEAR' method (RMSE\_L) from the scaling error by the 'CUBIC' method (RMSE\_C) for a series of 100 images (Fig. 1).



**FIG. 7.** Dependence of the scaling error RMSE by the 'LANCZOS4' method (RMSE\_LAN) from the scaling error by the 'LINEAR' method (RMSE\_L) for a series of 100 images (Fig. 1).

which are 9.661, 07.449, 9.370 and 5.643. The hypothesis of a normal distribution of RMSE values is confirmed for all methods, since for all methods  $χ2R < χ2T$ .

For each interpolation method, the average scaling error AR and the standard deviation σR for the RMSE values were calculated, and the limits of the confidence interval ARmin < AR < ARmax were calculated using formulas (3), (4) with a reliability of  $\gamma = 0.99$  (Table 1, Fig. 8).

Thus, based on the average scaling error  $A_R$  and taking into account the limits *ARmin* and *ARmax* of its confidence interval, the smallest scaling error is provided by the 'CUBIC' method, a slightly higher error by the 'LINEAR' method, an even larger error by the 'LANCZOS4' method, and the largest scaling error by the ' NEAREST' method.

**TABLE 1.** Average scaling error *A<sup>R</sup>* and standard deviation σ*<sup>R</sup>* for the RMSE values, limits of the confidence intervals  $A_{Rmin} < A_R < A_{Rmax}$  for a series of  $Q_{Im} = 100$  images (Fig. 1).

			<b>CUBIC LINEAR LANCZOS4 NEAREST</b>	
	$A_R$	0.075024 0.076572 0.080177		0.177774
	$\sigma_R$	0.030376 0.029270 0.032890		0.057526
		ARmin 0.067186 0.069020 0.071692		0.162933
		ARmax 0.082861 0.084124 0.088663		0.192616



**FIG. 8.** Average scaling errors *A<sup>R</sup>* and limits of its confidence intervals for  $Q_{lm} = 100$  images (Fig. 1).

With the 'LINEAR' method, the best scaling results are obtained for images with smooth changes in brightness and without clear contours. The Lanczos method allows to preserve the sharpness of edges and detail, which is useful for scaling images with a lot of detail (text, high-contrast graphic elements).

To compare the speed of the methods, the average scaling time  $A_T$  was determined, during which the scaling of a series of  $Q_{Im}$  images (Fig. 1) is performed by different methods (Table 2, Fig. 9). A computer with an AMD A4-6300 Processor, 3.70 GHz was used. The limits of the confidence intervals  $A_{Tmin} < A_T < A_{Tmax}$  with reliability  $\gamma = 0.99$  were also calculated.

In the process of determining the time of image interpolation, the scaling factor  $S_c = 8$  was used, so the total time of image scaling turned out to be significant. Due to this, it was possible to clearly divide the confidence intervals for the time of interpolation of images by different methods (Fig. 9). It was found that when scaling a series of investigated images, the smallest processing time was obtained for the 'LINEAR' method, and the largest time for the 'LANCZOS4' method.

Research results showed that bilinear interpolation algorithm has the highest speed, but in terms of accuracy it is slightly inferior to the bicubic interpolation algorithm due to partial image smoothing. Therefore, the bicubic interpolation algorithm can be used in cases where maximum scaling speed is required, or for processing images with smooth changes in brightness and without clear contours.

The Lanczos algorithm preserves edge sharpness and detail, which is useful for scaling images with a lot of detail. However, the Lanczos algorithm has the lowest speed, so it is not recommended to use it in high-speed systems.

**TABLE 2.** The average scaling time  $A_T$  for a series of  $Q_{lm} = 100$ images and the limits of the confidence intervals  $A_{Tmin} < A_T < A_{Tmax}$ ; image scaling factor  $S_c = 8$ .

		CUBIC LINEAR LANCZOS4 NEAREST	
$A_T$ , s	22.13042 16.34675 47.48684		19.19302
	$A_{Tmin}$ , s 21.71049 16.04037 46.85289		18.95186
	$A_{Tmax}$ , s 22.55036 16.65312 48.12078		19.43418



**FIG.** 9. Average scaling time  $A_T$  for a series of  $Q_{Im} = 100$  images and limits of its confidence intervals; quantity of scaling attempts  $Q_{Tm} = 9$ , image scaling factor  $S_c = 8$ .

The nearest neighbor algorithm has a significantly higher interpolation error compared to other algorithms, so this method is useful only for comparison or training purposes.

#### **V. CONCLUSION**

A study of the accuracy and speed of digital image scaling using interpolation algorithms was conducted. Algorithms of nearest neighbor, bilinear and bicubic interpolations, Lanczos are considered. Scaling accuracy is estimated through the RMSE between the pixel values of the original and scaled images.

The main scientific contribution of the authors is that, according to the Pearson's  $\chi^2$  test, the hypothesis of a normal distribution of RMSE values for all investigated image interpolation algorithms is confirmed. Due to this, for each interpolation method, not only its average scaling error *A<sup>R</sup>* is calculated, but also the bounds of its confidence interval *ARmin* and *ARmax* with a given reliability γ. Such an analysis of interpolation algorithms made it possible to more accurately assess their accuracy and speed, as well as advantages and disadvantages, areas of use and ways of improvement.

The research results showed that in most cases the smallest scaling error is provided by the bicubic interpolation algorithm, which is slightly inferior in speed to the nearest neighbor and bilinear interpolation algorithms. Therefore, in the general case, it is advisable to scale images using the bicubic interpolation algorithm.

The application of the considered interpolation algorithms leads to certain image distortions. Therefore, the use of artificial neural networks [14], in particular, convolutional neural networks, is promising for the highest quality image scaling.

#### **AUTHOR CONTRIBUTIONS**

S.B. – conceptualization, writing-review and editing, supervision:  $Yu.H$  – methodology, software, resources, writing-original draft preparation, visualization, validation, investigation.

#### **COMPETING INTERESTS**

The authors declare no conflict of interest.

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## **Аналіз результатів масштабування цифрових зображень алгоритмами інтерполяції**

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**АНОТАЦІЯ** Масштабування цифрових растрових зображень часто використовується в сучасних комп'ютерних і телекомунікаційних системах. Серед алгоритмів масштабування зображень найбільш поширеними є алгоритми інтерполяції, а саме найближчого сусіда, Ланцоша, білінійної та бікубічної інтерполяцій. Проте, в процесі масштабування зображень алгоритмами інтерполяції на них виникають характерні спотворення. Тому в даній роботі виконано програмну реалізацію та дослідження алгоритмів інтерполяції зображень з метою виявлення їх переваг і недоліків, сфер використання та шляхів удосконалення. З метою дослідження алгоритмів інтерполяції на основі початкового зображення fRGB виконано обчислення масштабованого зображення fRGBs, а потім на основі fRGBs обчислено масштабоване зображення fRGBs2 із розмірами початкового зображення. Точність масштабування оцінено через корінь середньої квадратичної помилки (root mean square error – RMSE) між значеннями пікселів початкового та масштабованого зображень. Програму для масштабування зображень розроблено на мові Python. Обчислення масштабованих зображень виконано функцією cv2.resize() бібліотеки OpenCV. За допомогою розробленої програми проведено масштабування серії з 100 зображень, досліджено точність та швидкодію масштабування цифрових зображень за алгоритмами інтерполяції. Для кожного метода інтерполяції обчислено середню помилку масштабування AR та межі її довірчого інтервалу ARmin та ARmax із заданою надійністю γ. Визначено середній час масштабування AT для серії з 100 зображень різними алгоритмами. Результати досліджень показали, що у більшості випадків найменшу помилку масштабування забезпечує алгоритм бікубічної інтерполяції, який незначно поступається за швидкодією алгоритмам найближчого сусіда та білінійної інтерполяції. Розроблено рекомендації по застосуванню алгоритмів інтерполяції. Показано, що для найбільш якісного масштабування зображень є перспективним застосування згорткових нейронних мереж.

**КЛЮЧОВІ СЛОВА** масштабування цифрових зображень, алгоритм найближчого сусіда, білінійна інтерполяція, бікубічна інтерполяція, алгоритм Ланцоша.



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