Security of Infocommunication Systems and Internet of Things

2023 Vol 1, No 1

https://doi.org/10.31861/sisiot2023.1.01003

Received 04 May 2023; revised 17 May 2023; accepted 26 June 2023; published 30 June 2023

Analysis of Self-Similar Binary Sequences

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ABSTRACT Rectangular pulses are simple to form, including with the help of a modern radio element base. In the work, a calculation analysis of the base and correlation coefficient of a binary sequence with a length of eight rectangular pulses was carried out. The obtained calculations of the base of sequences were analyzed, and a conclusion was made about which of them is the most suitable for encoding information during data transmission over the radio channel. Based on the calculations of the correlation coefficient between the series of pulse sequences, it was concluded which of them could be separated in the channel. Based on the structure of the self-similar sequence of pulses, a mathematical model and an expression of the spectral density of the proposed signal are written. A new method of correlation analysis for sequences that are symmetrical was also proposed, which allowed for improved recognition of the useful signal at lower signal-to-noise ratios in the communication channel. The method is a combination of autocorrelation and cross-correlation functions. The right and left halves of the symmetrical sequence and the reference signal are compared. The ratio of the height of the main petal to the side petals in our proposed correlation function is greater than in the classical version. To obtain the greatest possible ratio of the height of the petals, it is advisable to choose symmetrical sequences that have the largest base. With the help of such sequences, it is planned to encode one bit of information, which should improve the recognition of transmitted information against the background of noise. The obtained results are planned to be used for the development of a transmitter and receiver with an improved coding method for recognizing a useful signal against a background of noise.

KEYWORDS self-similarity, momentum, coding, correlation.

I. INTRODUCTION

Simple signals, such as rectangular pulses, can be used to encode information in simple radio systems for radio data exchange. They are easy to form and recognize with the help of a modern radio element base. A relatively unused class of signals are self-similar signals that can be used to transmit information in telecommunications [1, 2]. Such signals are characterized by broadband, and communication systems based on them are noise-resistant and safe [3, 4, 5]. The improvement of the coding method using self-similar signals should increase the reliability of the unambiguous selection of the appropriate codes against the background of interference of various origins. The paper [6, 7] shows the positive results of using such sequences.

The purpose of the work is to study the parameters of the proposed pulse sequences, highlight their advantages, and select the most suitable ones for coding according to the parameters of the base and correlation coefficient. Also analyze the effectiveness of a new method of correlation analysis for recognizing a useful signal against a background of noise.

II. ANALYSIS OF PARAMETERS OF BINARY SEQUENCES

The base of signals is one of their main characteristics, based on the value of which it is possible to conclude about the protection of information signals from the interference of various types. Broadband signals are those whose bandwidth coefficient (the product of the signal spectrum width and its duration $B = F \cdot Ts$) is greater than one [3].

$$B = F_{eff} \cdot T_S = F_{eff} \cdot \tau \cdot 2l \qquad (1)$$

where F_{eff} is the effective width of the signal frequency spectrum; T_S is the duration of a pulses sequence; $2l$ is the number of pulses in a sequence; τ is the duration of a pulse in a sequence (we took it equal to 1 µs).

When evaluating the basis of the offered signals, the width of their spectral band was determined by the noise equivalent method according to the ratio [8].

$$F_{eff} = \frac{1}{G_{\max}} \int_0^{\infty} G\left(f\right) df \, , \qquad (2)$$

where F_{eff} is the effective width of the signal frequency spectrum; $G_{\max}$ is the maximum value of the spectral density of the signal power; $G\left(f\right)$ is the spectral density of the signal. Let's create sequences with codes from 1 to 255 in binary format. The duration of one bit (one pulse) is taken to be 1 µs, so the duration of a series of pulses that follow each other continuously is taken to be 8 µs.

Let's calculate the basis for each pulse sequence. Calculations were performed using the MatLab software environment. The results of the calculations of the base of sequences are presented as a graph in FIG. 1. The abscissa shows the number of the binary sequence and the ordinate shows the base.

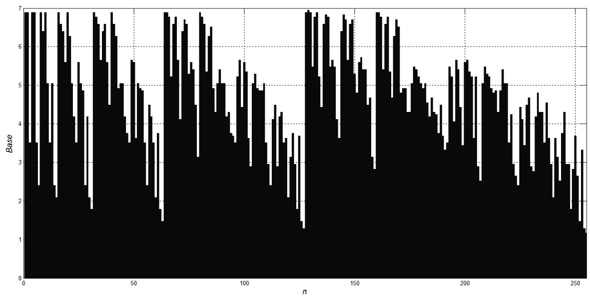


FIG. 1. Results of sequence database calculations.

It is obvious that for the zero and the last combination the spectrum will be narrower than in other cases. The effective frequency band and the base will be smaller because in the first case there is no signal, and in the second case we will receive one pulse.

Analyzing the obtained calculations, we can conclude that the sequences with the codes are the most suitable for encoding information when transmitting data over a radio channel: 1,2,4,5,8,9,10,16,17,18,20,21,32,33,34,36,37,40, 41,42,64,65,66,68,69,72,73,74,80,81,82,84,85,128,129, 130,132,133,136,137,138,144,145,146,148,149,160,161, 162, 164,165,168,169,170 (1, 2, 4, 8, …, 2^N, where $N = 0,1,2,...$ and 3, 5, 6, 9, 10, …, $2^N + 2^M$, where $M = 0,1,...,5$). The remaining sequences have the smallest base.

III. CORRELATION COEFFICIENT OF BINARY SEQUENCES

Let's also calculate the correlation coefficient between sequences according to formula (3), which will allow us to draw a conclusion about the possibility using them in parallel transmission channels or better extraction from noisy signal. We will present the results of the calculations as a planar graph (FIG. 2).

$$r = \frac{\int_a^b f(t)\, g(t)\, dt}{\sqrt{\int_a^b f^2(t)\, dt}\sqrt{\int_a^b g^2(t)\, dt}} \qquad (3)$$

where $f(t)$ and $g(t)$ are the studied signals.

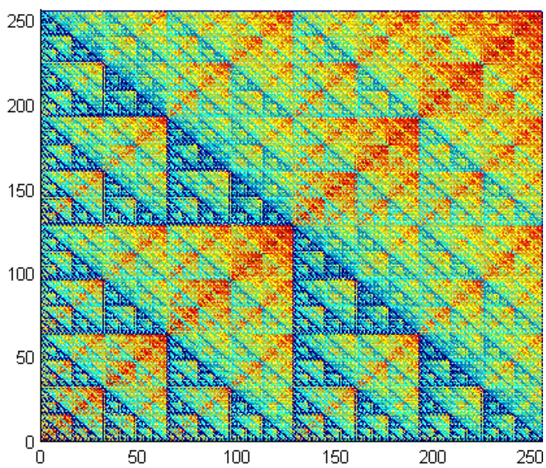


FIG. 2. Dependence of the correlation coefficient between sequences.

FIG. 2 shows sequence numbers along the axes. Dark points correspond to a correlation coefficient equal to zero. Analyzing the correlation coefficient between a series of pulse sequences, the following pairs of sequences can be used to separate channels or uniquely separate signals when they transmit different bits 0 and 1 in different sequences: 1.2; 1.4; 1.6; and others between which the correlation coefficient is zero.

IV. SELF-SIMILAR PULSE SEQUENCES

There are known works in which it is offered to form pseudo-random sequences of pulses [9]. We suggest applying a sequence of pulses that follow each other continuously (FIG. 3). These sequences are symmetrical, which should allow comparing the received signal with itself.

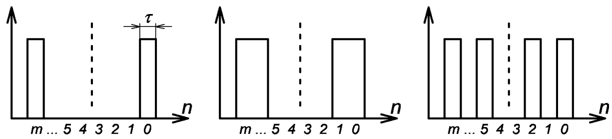


FIG. 3. Examples of pulse sequences used in the "Smart House" system under development; *m* where is the number of pulses in the sequence, *n* is the pulse number in the sequence, *τ* is the duration of the pulse.

The sequence of pulses is formed in such a way that the older and younger halves of the pulses are mirror-symmetrical to each other. First, we set the number of pulses *l* in the younger half of the sequence – we will get 2^l possible combinations in binary form. Then we mirror the given combination and supplement it with the initial one and get the designed sequence with a length $2l$ of pulses. The pulses follow one another continuously, their amplitude is the same. The sequences themselves during transmission are separated by time slots.

Let's form symmetrical sequences of pulses with a length of 8 bits. The older half of the sequence is a symmetrical reflection of the younger one. An example of the calculation window is presented in FIG. 4. In this window, we can see the waveform, spectrum and power spectrum.

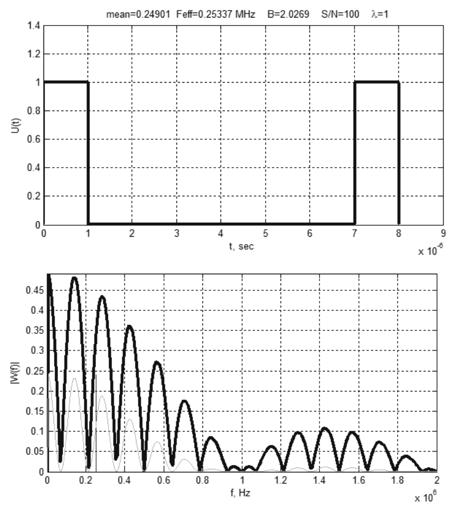


FIG. 4. Example of the calculation result window in the MatLab software environment.

From the obtained set of binary sequences (item 1), we will select such binary codes in which the youngest and the oldest half-bytes (nibbles) are mirror-imaged (symmetric). There are 15 such codes: 24, 36, 60, 66, 90, 102, 126, 129, 153, 165, 189, 195, 219, 231, 255. Based on them, we will form sequences of pulses and calculate their bases (FIG. 5).

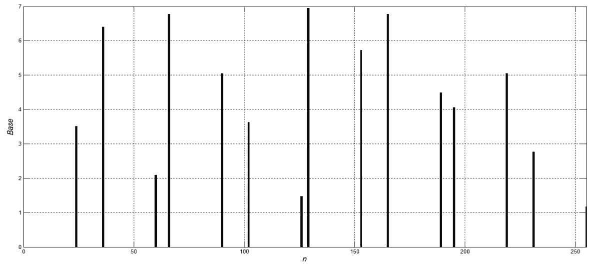


FIG. 5. Results of sequence database calculations.

As can be seen from the figure, sequences based on codes 36, 66, 129, 165 have the largest base. We will use the symmetry of pulses in the correlation analysis as one of the signs of this pulse.

V. MATHEMATICAL MODEL OF A SELF-SIMILAR SIGNAL

Based on the structure of a self-similar sequence of pulses, the mathematical model can be written as a sum of sequences of pulses with the same amplitude [10]:

$$x(t)=A\sum_{k=0}^{7}\left[Q(t-t_k)-Q(t-t_k-\tau)\right], \quad (4)$$

where A is the pulse amplitude; k is the sequence number of the pulse; $Q(t)$ is the Heaviside function; τ is the pulse duration (the same for all pulses).

Using the expression of the time dependence of the signal, it is possible to calculate the expression of the spectral density of the offered fractal by finding the Fourier transform. The complex spectral density of such a signal is equal to:

$$S(\omega)=\frac{2A}{\omega}\sin\left(\frac{\omega\tau}{2}\right)\cdot e^{-j\frac{\omega\tau}{2}}\sum_{k=0}^{7}x_k e^{-jk\omega\tau}, \quad (5)$$

where x_k is the value of the bit in the sequence (0 or 1); k is the pulse number in the sequence; A is the pulse amplitude; τ is the pulse duration.

The modulus of the complex spectral density is defined as:

$$|S(\omega)|=2A\left|\frac{\sin(\omega\tau/2)}{\omega}\right|\cdot\sqrt{\left[\sum_{k=0}^{7}x_k\cos(k\omega\tau)\right]^2+\left[\sum_{k=0}^{7}x_k\sin(k\omega\tau)\right]^2} \quad (6)$$

The argument of the complex spectral density is equal to:

$$\phi(\omega)=-arctg\left[\sum_{k=0}^{7}x_k\sin(k\omega\tau)\Big/\sum_{k=0}^{7}x_k\cos(k\omega\tau)\right].$$

VI. A METHOD OF RECOGNIZING A SEQUENCE OF PULSES AGAINST A BACKGROUND OF NOISE

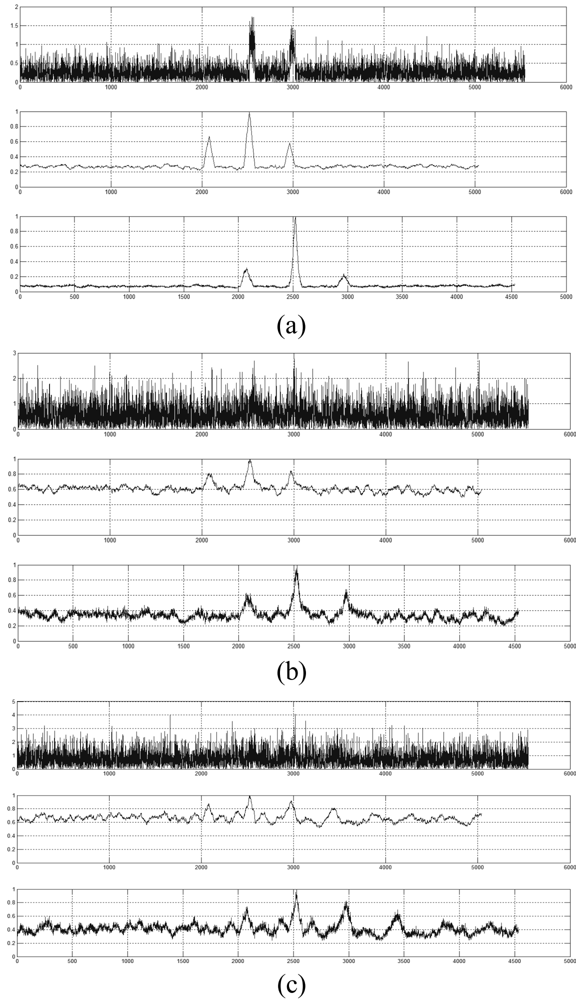
The method is a combination of autocorrelation and cross-correlation functions. The right and left halves of the symmetrical sequence and the reference signal are compared. The formula, according to which the method works, looks like this:

$$K(n)=\sum_{n=0}^{3}s_{[0\ldots m]}(n)\cdot s_{[(2m-1)\ldots m]}(m-n)\cdot s_{op[0\ldots m]}(n), \quad (7)$$

where n is the bit number in the sequence; $s_{[0\ldots m]}(n)$ is the bits in a sequence with numbers from 0 to m; $s_{[(2m-1)\ldots m]}(m-n)$ is the symmetrically placed bits in sequence with numbers from $2m-1$ to m; $s_{op[0\ldots m]}(n)$ is the reference data (initial signal).

In this formula, the autocorrelation and cross-correlation functions are calculated at the same time - it is their combination. This should give a gain in extracting pulse sequences from noise.

FIG. 6 presents the results of calculations according to the formula (7) at different ratios of noise power to signal power. Calculations were performed for one of the pulse sequences. In the figures, the upper graph shows the signal against a background of noise, the middle one shows the mutual correlation function, and the lower one shows the correlation function we offered.



(a) (b) (c)

FIG. 6. Results of correlation function calculations at different ratios of noise power to signal power: the upper graph in the figures is the signal, the middle one is the mutual correlation function, and the lower one is the offered function. The ratio of signal power to noise power: a) 10 dB, b) 3 dB, c) 0 dB.

As can be seen from the graphs, the ratio of the main petal to the side petals in our offered correlation function is greater than in the classical version. As shown by the calculations of the classical correlation function and the offered one, the ratio of the "height" of the main petal to the "height" of the lateral ones at different signal/noise ratios is: 1.82 (classical), 4 (offered) at 10 dB; 2 (classic), 2.75 (suggested) at 3 dB; 1.25 (classic), 2 (suggested) at 0 dB. In this way, we can recognize signals even when the ratio of signal power to noise power is 0 dB.

VII. CONCLUSION

The calculations of the signal base and correlation coefficient made it possible to choose the most optimal sequences of pulses for encoding signals. The results of calculations according to the offered correlation function at different ratios of noise power to signal power showed that it is possible to recognize the offered pulse sequences at a power ratio of up to 0 dB. When using the generally accepted mutual correlation function, there is no unambiguous recognition of the signal under such conditions.

AUTHOR CONTRIBUTIONS

A.V. – conceptualization, methodology, investigation; R.P. – conceptualization, methodology; M.R. – writing (original draft preparation), H.L. – writing (review and editing).

COMPETING INTERESTS

The authors declare no competing interests.

REFERENCES

[1] B.B. Mandelbrot. *The Fractal Geometry of Nature.* Hardcover: Times Books, 1982, 468p.

[2] J. Feder *Fraktals*. Springer, 1988, 310p.

[3] V.N. Bolotov, S.E. Kolesnikov, Yu.V. Tkach, Ya.Yu Tkach, P.V. Khupchenko. "Fractal Communication System", *Electromagnetic Phenomena*, vol.7, №1 (18), 2007.

[4] O.B. Novikova "Fractal spline model of a broadband signal", *Bulletin of the National University "Lviv Polytechnic". Radio electronics and telecommunications*, no.738, pp.28–33, 2012.

[5] Yu. Bobalo, A. Veryha, M. Klymash, B. Mandziy, R. Politanskyi "Design and Hardware Implementation of Fractal Comb-Structured Signals", *Smart Computing Review*, vol.4, no.6, pp.459–469, Dec. 2014.

[6] A.D. Veryha, R.L. Politanskyi, M.S. Rusnak, N.Yu. Semenyuk "Self similar sequences of impulses and methods of their detection", *VII International scientific and practical conference "Physical and technological problems of information transmission, processing and storage in information communication systems"*, Chernivtsi, November 8-10, 2018, pp.95–96.

[7] S.A. Ostanyn "Fractal noise generator of arbitrary size", *Journal of radio electronics*, no.8, 2012.

[8] V.N. Bolotov, Yu.V. Tkach "Extraction of fractal signals in a complex electromagnetic environment", Electromagnetic phenomena, vol.3, No. 2 (10), pp.211–227, 2003.

[9] S.V. Saveliev "A countable set of binary sequences for broadband communication systems based on a system with dynamic chaos", *111 All-Russian Conference "Radiolocation and Radio Communication" - IRE RAS*, October 26-29, 2009, pp.488–493.

[10] Yu.Ya. Bobalo, R.I. Zhelyak, M.D. Kiselichnyk, Z.O. Kolodiy, B.A. Mandziy, V.M. Yakubenko; under the editorship Dr. Tech. Sciences, Prof. Yu.Ya. Bobalo and Dr. Tech. Sciences, Prof. B.A. Mandziy *Mathematical models and methods of analysis of electronic circuits: teaching. Guide*. Lviv: Publishing House of Lviv Polytechnic, 2013, 320p.

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Дослідження самоподібних двійкових послідовностей

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АНОТАЦІЯ Прямокутні імпульси є простими для формування, в тому числі за допомогою сучасної радіоелементної бази. В роботі проведено розрахунковий аналіз бази та коефіцієнту кореляції двійкової послідовності довжиною вісім прямокутних імпульсів. Проаналізовано отримані розрахунки бази послідовностей та зроблено висновок, які з них є найбільш придатними для кодування інформації при передаванні даних по радіоканалу. На основі розрахунків коефіцієнту кореляції між серіями послідовностей імпульсів зроблено висновок котрі з них можна розділити в каналі. Виходячи зі структури самоподібної послідовності імпульсів записана математична модель та вираз спектральної густини пропонованого сигналу. Також запропоновано новий спосіб кореляційного аналізу для послідовностей, які є симетричними, що дозволило покращити розпізнавання корисного сигналу при менших відношеннях сигнал/шум в каналі зв'язку. Метод полягає в поєднанні автокореляційної та взаємної кореляційної функцій. Порівнюються права і ліва половини симетричної послідовності та опорний сигнал. Відношення висоти головної пелюстки до бокових в запропонованій нами кореляційній функції є більшим ніж в класичному варіанті. Можемо розпізнавати сигнали при відношенні потужності сигналу до потужності шуму 0 дБ. При застосуванні загальноприйнятої взаємної кореляційної функції однозначного розпізнавання сигналу при таких умовах не має. Для отримання якнайбільшого відношення висоти пелюсток доцільним є вибрати симетричні послідовності, які володіють найбільшою базою. За допомогою таких послідовностей планується кодувати один біт інформації, що повинно підвищити розпізнавання переданої інформації на фоні шумів. Отримані результати планується використати для розроблення передавача та приймача з покращеним способом кодування для розпізнавання корисного сигналу на фоні шумів.

КЛЮЧОВІ СЛОВА самоподібність, імпульс, кодування, кореляція.