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Comparative Analysis of Optimal Power Flow Algorithms in Networks with Distributed Generation

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ABSTRACT This paper presents the formulation of the optimisation problem and a comparative analysis of real-time optimal power flow (RT-OPF) algorithms for distribution networks with a high penetration of distributed energy resources (DERs), including solar and wind generation as well as battery energy storage systems. The intermittent nature of many DERs and the emergence of bidirectional power flows increase the risk of violations of operational constraints, primarily voltage deviations and overloading of network elements. This creates a need for fast DER coordination mechanisms capable of operating on a sub-second timescale under limited communication bandwidth. The RT-OPF problem is formulated as a constrained optimisation problem in which the active and reactive power setpoints of DERs are selected to minimise aggregate undesirable effects and operating costs, subject to compliance with the local technical constraints of the devices and the network operating limits. Three algorithmic approaches widely described in the literature are considered: the primal-dual gradient method (PDGM), the alternating direction method of multipliers (ADMM), and proximal atomic coordination (PAC). The comparison is carried out according to five practically important criteria: the number of iterations per control interval, the complexity of local computations, communication requirements and scalability, robustness to delays and partial asynchronicity, and the ability to ensure compliance with local and network constraints in closed-loop control. It is shown that PDGM is generally the most suitable baseline choice for sub-second RT-OPF in large-scale aggregator-based systems owing to the simplicity of its local updates and its compact data exchange, whereas PAC provides smoother behaviour near active constraints at the cost of a more computationally demanding local step. ADMM can achieve high consensus accuracy given sufficient iterations; however, in practice it is more appropriate for slower control loops and in the presence of a reliable synchronous communication infrastructure.

KEYWORDS optimal power flow, distributed energy resources, distributed optimisation, distribution networks, closed-loop control.

I. INTRODUCTION

The rapid growth in the share of distributed energy resources (DERs) is fundamentally reshaping the architecture of electric power systems, transforming historically passive distribution networks into active multi-node systems with bidirectional power flows [1]. In the context of the digital transformation of Ukraine's energy sector and the development of Smart Grid concepts, this process creates a demand for advanced real-time control algorithms to ensure system flexibility and resilience [2].

With the increasing penetration of DERs – including photovoltaic and wind generation, energy storage systems, and controllable loads such as electric vehicles – the risk of violating operational constraints in the distribution network rises. This specifically concerns voltage deviations and equipment overloads, necessitating the implementation of active regulation strategies [3, 4]. In practical systems, this requires algorithms capable of coordinated control over a large number of DERs on a sub-second timescale, while operating under limited data exchange between the DERs and their aggregator.

The variability of solar and wind power output leads to rapid changes in the system state. Consequently, there is a need for algorithms that do not merely solve a static problem but can track a time-varying optimum in real time [5], ensuring network flexibility and stability amidst continuous disturbances [3, 6].

For domestic distribution networks, the tasks of voltage profile optimisation, loss minimisation, and ensuring compliance with operational constraints under mass DER integration are of high applied relevance for distribution system operators (DSOs) and flexibility stakeholders, such as aggregators and DER owners [7].

In this context, the optimal power flow (OPF) problem consists of determining the active and reactive power setpoints for controllable DERs that minimise the total undesirable effects and costs (e.g. technical network losses and penalties for deviations from the desired operating mode), subject to all necessary constraints (voltage limits, thermal limits of network components, inverter capability limits, etc.). Traditional centralised approaches to solving the OPF problem involve aggregating all information at a single control centre. However, in large-scale systems, such solutions face critical communication latencies, scalability issues, and single-point-of-failure risks. As an alternative, the transition to coordinated decentralised optimisation methods has become highly relevant [8], as they enable distributed computing across nodes and minimal data exchange with guaranteed performance.

The aim of this paper is to select an algorithmic approach for solving the coordinated decentralised real-time optimal power flow (RT-OPF) problem. The scientific novelty of this work lies in the comprehensive systematic comparison of three algorithms tailored for sub-second RT-OPF in distribution networks with a high penetration of DERs, based on a unified set of engineering-significant criteria. The practical value consists in formulating recommendations for algorithm selection depending on speed, communication overheads, latency robustness, and available computational resources. This article is analytical and comparative in nature and is based on the synthesised results reported in the literature.

II. OPTIMISATION PROBLEM FORMULATION

A. Local technical constraints of DERs. Consider a distribution network in which DERs, such as photovoltaic (PV) systems, wind turbines (WTs), and battery energy storage systems (BESSs), are connected to network nodes.

Let the set of all DERs controlled by the aggregator be denoted by $I = \{1,2, \dots, N\}$. If necessary, the aggregator's control area may be decomposed into subsets corresponding to individual feeders or control zones.

For each DER, a local controller (DER agent), i.e., a hardware-software control module located either at the device side or at the edge level, determines the setpoints of active and reactive power, denoted as P_i and Q_i, respectively. These values are treated as the setpoints to be implemented by the corresponding DER during the next control interval. Here, i denotes the index of a specific DER from the set I of all DERs ($i \in I$).

Furthermore, each DER is subject to its own local technical operational constraints (capability limits), specifically regarding:

- maximum and minimum active power;
- reactive power;
- apparent power;
- power ramp rates;
- state-of-charge (SoC) limits for BESS.

Consequently, the algorithm must not assign setpoints to the DERs that they are technically unable to implement. In other words, every solution to the optimisation problem must belong to the feasible control set of the specific device. The requirement for DERs to operate within certain local technical limits can be expressed mathematically as follows:

$$(P_i,\ Q_i) \in\ Y_i,\ \ \forall i\ \in\ I, \tag{1}$$

where Y_i represents the active and reactive power ranges of the DER that satisfy the local technical constraints within which the power outputs of each DER must remain, as determined by its type and characteristics [9].

B. System-level network constraints. Since DERs are connected to a common electrical network, their setpoints P_i and Q_i are interdependent through the network operating state. In other words, the combined effect of multiple DERs may result in a situation where an operating point that is locally feasible for an individual DER becomes unacceptable for the network as a whole. Therefore, in addition to the local technical constraints of DERs, system-level network constraints must also be considered. These are determined by regulatory documents, grid codes, and the technical characteristics of network equipment [9, 10]. Such constraints typically include:

1. Node voltage constraints. The voltage at each node of the distribution network must remain within the limits established by regulations. For low- and medium-voltage networks under normal operating conditions (according to DSTU EN 50160, for 95% of 10-minute average values over a one-week period), a typical requirement is to maintain the RMS voltage within the range of -10% to +10% of the nominal value [10]. However, in active DER control tasks, internal (operational) limits that are stricter than regulatory ones are often applied as an engineering margin for the closed-loop control [5, 11]. Regulatory voltage limits are defined by national power quality standards, network codes, and the technical requirements of the DSO. Operational limits are introduced as an engineering margin within the control loop in order to enhance reliability and operating stability. The general purpose of such constraints is to ensure the proper operation of consumers' equipment, prevent equipment damage, and maintain system stability.

2. Constraint on total power at the feeder head (at the point of connection to the substation/transformer).The power at the feeder head is limited by the technical characteristics of the equipment and by operating requirements, namely the transformer rated power, the permissible ampacity of the lines, the thermal withstand capability of the equipment, and any contractual or operational limits imposed by the DSO [12]. This constraint is formalised as expression:

$$P_{min}^{f} \le P_0 \le P_{max}^{f}\ , \tag{2}$$

where P_0 is the active power at the feeder head, i.e. the total active power transferred between the substation and the feeder; P_{min}^{f} та P_{max}^{f} are the lower and upper permissible limits of the active power at the feeder head, respectively. These depend on the transformer rated power, the permissible overload factor, and ambient temperature conditions. In practice, this limit is set by the DSO on the basis of the technical characteristics of the equipment.

3. Constraints on line and transformer overloading. Every line and transformer has rated current and power limits determined by the thermal withstand capability of the equipment and permissible overload conditions. Typically, these constraints are expressed as follows:

$$\begin{gathered} |\ I_{line}\ | \le I_{max}, \\ S_{trans} \le S_{rated}, \end{gathered} \tag{3}$$

where I_{line} is the current in the corresponding network line. This generally refers to the RMS current for a specific phase or the maximum current across phases. The absolute value of the current is evaluated, meaning the constraint applies to the magnitude of the current regardless of the flow direction; I_{max} is the maximum permissible current for the line, determined by the conductor/cable cross-section, insulation type, installation method, and ambient temperature conditions; S_{trans} is the actual transformer loading in terms of apparent power (i.e. the total load considering both active and reactive components); S_{rated} is the transformer rated apparent power specified by the manufacturer, i.e. the limit at which the transformer can operate under normal conditions without overheating.

These values are determined by equipment data sheets, manufacturer standards, permissible thermal load calculations, and DSO operating instructions. Violating these limits can lead to overheating, accelerated insulation degradation, and emergency shutdowns. Consequently, even if each DER operates within its individual limits, the aggregate action of all devices must not create hazardous operating conditions in the network. Such constraints in an optimisation problem can be both static and dynamic in nature. In general, the requirements for fulfilling systemic network constraints that must be satisfied by the algorithm at any given time can be expressed mathematically as:

$$\underline{G}_j \leq g_j(\{P_i,\ Q_i\}_{i\in \mathrm{I}}) \leq \overline{G}_j,\ \ \forall j\ \in\ J, \tag{4}$$

where j is the index of a specific constrained measurement from the set of all constrained measurements J; $\underline{G}_j$ and $\overline{G}_j$ are the lower and upper permissible bounds of the constrained measurement j, respectively; $g_j(\{P_i,\ Q_i\}_{i\in \mathrm{I}})$ is a function that makes it possible to approximately represent the value of any constrained measurement as a function of the power injected into the network by the DERs [5, 11, 13, 14].

C. Minimisation of costs and undesirable effects. In addition to satisfying the local technical constraints of DERs and the system-level network constraints, the optimisation problem must also include a criterion for operational efficiency. In practical terms, this means that, among all feasible sets of setpoints (P_i, Q_i), the optimisation algorithm should select the one that minimises the aggregate undesirable impact on the network and equipment and/or the associated economic and operational costs [11, 12, 15]. In this study, costs and undesirable effects are not treated as a single value but as a set of components relevant to different participants in the control process, including DER owners, the aggregator, and the DSO. The main components of this criterion are outlined below:

1. Electrical network losses. Electrical network losses refer to the active power losses occurring in lines and transformer windings due to current flow. An increase in current leads to higher thermal losses, greater conductor heating and, consequently, reduced energy efficiency in transmission and additional operational costs for the DSO. The algorithm can reduce such losses because the control of DER active and reactive power directly affects current magnitudes and internal power flows within the network [12, 13]. Thus, it becomes possible to limit excessive reactive power circulation or to obtain such a distribution of active power as minimises the need to transfer energy over long feeder distances, thereby reducing currents in long sections and the associated losses.

2. Forced curtailment of renewable generation. For PV systems and WTs, there exists at each moment a potentially available active power determined by the prevailing meteorological conditions. However, network constraints, for example voltage limits, equipment loading limits, or feeder-head limits, may require the active power setpoint to be reduced below the available level in order to maintain an acceptable operating condition. Such a reduction is commonly referred to as curtailment [5, 11]. The undesirability of this phenomenon lies in the fact that the actual electricity output from renewable energy sources is reduced, causing lost revenue for the owner. In a broader sense, it also lowers the share of renewable generation that is effectively integrated into the power system.

3. Operating costs and battery degradation. For BESSs, undesirable effects are associated primarily with battery degradation under intensive or irrational use. In particular, the use of BESSs for frequent short-term operating-point corrections may accelerate degradation processes and thereby affect the available capacity and service life of the system. In practice, degradation depends on the number of charge/discharge cycles, depth of discharge, charging/discharging current magnitude, operation at undesirable SoC levels, and temperature conditions. Accordingly, the optimisation formulation should ensure a compromise between the short-term benefit of engaging the BESS in operating-point control and the long-term preservation of its useful life, avoiding operating modes that lead to accelerated degradation [5, 11].

4. Smooth control action and avoidance of frequent switching. Even when all strict local technical and system-level network constraints are satisfied, excessively frequent or abrupt changes in active and reactive power setpoints are undesirable from an operational perspective. In particular, such control dynamics may lead to increased wear of power-electronic components, larger voltage fluctuation amplitudes, deterioration in power quality indices, and undesirable frequent transitions between charging and discharging modes of storage systems. For this reason, it is advisable to include in the optimisation problem a component of the criterion that regularises the control trajectory and promotes smooth setpoint variations. This component is particularly important in real-time operation, where setpoints are updated frequently [5, 11].

5. Penalties for undesirable operating modes and service requirement violations. In addition to strict physical constraints, which must always be satisfied, practical DER control systems often define additional operational objectives that reflect preferred operating modes and reliability margins. Such objectives may include, for example, avoiding prolonged operation in close proximity to voltage limits, limiting sustained high transformer loading, maintaining operation within a preferred working range even when formal strict limits have not yet been violated, and minimising deviation from commitments made to the aggregator. These requirements are best represented not as additional strict constraints, but as penalty terms in the optimisation objective function (cost function). In this case, undesirable operating conditions are not completely prohibited, but they incur a higher penalty, so that, all else being equal, the algorithm favours more reliable and preferable operating modes, thereby creating a controlled compromise between efficiency and operational reliability [5, 11, 15]. In general, the cost minimisation requirements can be expressed mathematically as:

$$\min_{\{P_i,\ Q_i\}_{i\in \mathrm{I}}} \textstyle\sum_{i\ \in\ \mathrm{I}} f_i(x_i), \tag{5}$$

where $x_i = (P_i, Q_i)$ represents the vector of active and reactive power setpoints selected by the DER controller for the next control interval. Components of the argument x_i carry distinct physical meanings depending on the specific infrastructure element. For PV systems and WTs, P_i represents the adjusted active power output (which can be actively curtailed relative to the maximum available meteorological power), and Q_i is the reactive power injected or absorbed by the inverter for voltage regulation. For BESSs, P_i denotes the charging power ($P_i < 0$) or discharging power ($P_i > 0$), while Q_i represents the reactive power support provided to the network node; $f_i(x_i)$ is the local cost function for the i-th DER, defined at the level of its control agent and representing the costs and undesirable effects associated specifically with its operation. The functions $f_i(x_i)$ are not universal and may differ among DERs depending on their nature and operating conditions [5, 11, 15].

D. Generalized optimisation problem formulation. In summary, it is evident from the foregoing that the optimisation problem consists in finding such a control vector of active and reactive power for each DER as satisfies all local technical constraints of the DERs, ensures the safe and compliant operation of the network, and minimises the aggregate performance criterion [11, 12, 15].

Accordingly, the problem of coordinated control of DER active and reactive power is formulated as the minimisation of an aggregate cost function subject to local technical and system-level constraints, as given by Eqs. (1), (4), and (5). A fundamental feature of this problem is that it must be solved not as a one-off task, but continuously over time, since DER operating conditions (available generation, SoC, charging/discharging state) and network operating conditions (voltage, power flows, loading) vary over timescales ranging from fractions of a second to minutes [5, 11].

E. Requirements for a real-time algorithm. In the classical OPF formulation, the system state is assumed to be conditionally ‘static’ within a single optimisation run. That is, data is first collected, then the optimal solution is computed, and only afterwards it is applied. However, in distribution networks with a large number of DERs, this logic often fails [5, 11], since the operating state of the network may change more rapidly than a centralised computation can be completed. For example, PV generation levels can fluctuate rapidly due to cloud cover [9].

Another crucial aspect is that, when hundreds or thousands of DERs are involved, substantial communication constraints arise, because transmitting the full state of each DER to a single centre and sending control commands back created delays and increases the risk of de-synchronisation [16, 17]. For this reason, an RT-OPF algorithm should be assessed not only in terms of its "mathematical accuracy", but also in terms of its ability to maintain the network within an admissible operating region under closed-loop control [5, 11]. The algorithm should satisfy the following criteria:

1. Performance [5, 11]. The performance requirement implies that one complete algorithmic step, encompassing local computations at the DER side, coordination by the aggregator, and communication exchange, should be completed within milliseconds or tens of milliseconds. This makes it possible to implement an overall control cycle within a few tenths of a second, depending on the configuration of the measurement infrastructure and communication channels. A fundamental feature of RT-OPF is that stable operation often does not require a perfectly accurate optimum; rather, it is sufficient to obtain an acceptable step in the direction of improving the objective function. Since the physical state of the system changes continuously, a strategy of rapid iterative tracking of the optimum is considerably more effective than prolonged computation of an exact solution for an operating condition that will already have become outdated by the time the computation is finished.

2. Minimal communication requirements and robustness to delays. The algorithm should restrict data exchange to the transmission of compact coordination signals only, such as local gradients or dual variables, thereby avoiding the need to exchange complete information on network topology or parameters [5, 11, 16]. This approach makes it possible to reduce significantly the burden on communication channels and ensures that the system remains operational under conditions of substantial latency and asynchronous data updates [18], which are typical of heterogeneous Smart Grid communication environments [19].

3. Scalability [15-17]. As the number of DERs subject to control tends to increase, the selected algorithm should exhibit linear $O(N)$ or quasi-linear $O(N\log N)$ computational complexity. The use of methods that require the solution of computationally heavy global subproblems, such as the inversion of large matrices or complex quadratic programming at each step, severely limits the scalability of the system. The algorithm should therefore distribute the main share of the computational burden to the edge level, that is, to local DER controllers, while maintaining minimal dependence on a central optimiser. This ensures stable system operation even when the network expands significantly.

4. Guaranteed convergence and stability [5, 11, 20]. Since RT-OPF operates as a closed-loop control algorithm based on current measurements, the selected method should guarantee a stable iterative process, preventing divergence of the computations and the emergence of undesirable oscillations in network operating parameters. For problems possessing convexity properties, it is critically important to have formally justified convergence guarantees, in particular the ability of the algorithm to converge to a neighbourhood of the optimal solution or of a saddle point, even under noisy measurement conditions [5]. In addition, the algorithm should provide mechanisms for the controlled tuning of parameters, such as update step sizes and regularisation coefficients, making it possible to achieve the required balance between the speed of optimum tracking and solution accuracy under real operating conditions [15].

Thus, the requirements for an RT-OPF algorithm follow directly from the physical and infrastructural constraints of DER control systems in real distribution networks, namely fast operating-state dynamics, limited communications, a large number of agents, and unavoidable asynchronicity [5, 11, 17, 18]. To solve the formulated optimisation problem, the literature considers a number of decentralised algorithmic approaches that satisfy these requirements to different degrees. In this paper, attention is focused on the three most relevant approaches: the primal-dual gradient method (PDGM), the alternating direction method of multipliers (ADMM), and proximal atomic coordination (PAC).

III. PRIMAL-DUAL GRADIENT METHOD

The PDGM is one of the fundamental approaches to solving constrained convex optimisation problems in a distributed formulation [15, 20]. In the RT-OPF context, this method is attractive because the local computations performed on the side of each DER are reduced to a simple "small-step" update of setpoints, while coordination with network constraints is achieved through dual variables. These variables are interpreted as "prices" or penalties for violations of voltage limits, feeder limits, and other operational constraints [11, 16]. Such a mechanism makes it possible to operate in a closed-loop control: at each control interval, the system does not necessarily compute the exact optimum, but it does stably adjust the operating mode towards a feasible and efficient region [5, 11].

The method is based on a trade-off between two components:

1. The local objective of the DER, which reflects undesirable effects minimisation for the specific device (for example, minimising forced power curtailment of a solar installation below the available level, limiting the intensity of BESS utilisation in order to preserve its lifetime, or avoiding abrupt changes in setpoints) [11, 16];

2. The coordination signal, formed on the basis of network operational constraints, which reflects the impact of DER setpoints on the feasibility of the operating mode and guides the aggregate action of the DERs towards a secure operating condition [11, 16].

The coordination signal is determined by the aggregator on the basis of measurements and/or state estimates of the distribution network within the relevant feeder, in particular voltages at monitored nodes, loading of network elements, and power at the feeder head. If the current operating state approaches the prescribed limits (for example, the upper or lower voltage bound, the feeder-head power limit, or the thermal limits of lines and transformers), the corresponding dual variables (or penalty coefficients) increase, and the coordination signal strengthens the incentive for the DERs to modify their active and reactive power setpoints so as to return the operating state to the feasible region [11].

A key feature is that the coordinator does not directly assign exact values of active P_i and reactive Q_i power to each DER. Instead, it transmits a compact coordination signal that determines the direction of setpoint adjustment, while each DER independently selects an admissible change within its own technical constraints [11, 16].

The local update of the setpoints of the i-th DER in the PDGM may be written as a gradient step followed by projection onto the feasible region [15, 20]:

$$x_i^{k+1} = Proj_{Y_i}\left(x_i^k - \alpha_i\left(\nabla f_i^k(x_i) + s_i^k\right)\right), \qquad (6)$$

where k is the iteration index of the algorithm, and $k+1$ denotes the subsequent iteration; x_i^k is the setpoint vector of the i-th DER at iteration k; x_i^{k+1} is the updated value of the DER setpoint vector at the next algorithmic step; $Proj_{Y_i}$ is the projection operator onto the set of feasible operating conditions Y_i, which ensures compliance with local technical constraints. It is assumed that Y_i is a closed convex set, in which case the projection operator is uniquely defined [15, 20]; $f_i^k(x_i)$ is the cost function at iteration k, reflecting the change in local undesirable effects of the i-th DER when its setpoints are varied, and $\nabla f_i^k(x_i)$ is its local derivative (gradient) which mathematically represents the marginal costs or sensitivities specific to the device type defined in the previous section (such as renewable power curtailment penalties, BESS degradation rates, and costs associated with deviations from preferred DER operating points); s_i^k is the coordination vector at iteration k, formed by the aggregator on the basis of measurements and network operational constraints [11, 16]; α_i is the step-size parameter that determines the intensity of setpoint adjustment [15, 20]. The larger its value, the larger the "jumps" that the algorithm permits in changing the setpoints.

This representation highlights a practically important property of the method: even if the unconstrained gradient step leads to values that are physically unattainable or inadmissible for a particular DER, the projection operator $Proj_{Y_i}$ guarantees the selection of the nearest feasible operating point within Y_i, thereby ensuring the technical implementability of the resulting setpoints [15, 20].

The coordination signal s_i is a generalised coordination signal formed by the aggregator on the basis of current measurements or estimates of the state of the distribution network, in particular voltages at monitored nodes, loading of network elements, and power flows at the feeder head, together with linearised sensitivities describing the dependence of these operating quantities on changes in the active and reactive power of the DERs [11, 16]. In this sense, s_i reflects the extent to which a change in the setpoints (P_i, Q_i) of a particular DER affects the proximity of the system to its operational constraints.

In practical terms, the components of s_i may be interpreted as "local marginal penalties" or prices for an incremental increase in active or reactive power at the point of connection of the given DER. If a violation is observed, or if the operating state approaches the voltage limits, the feeder-head power limits, or the thermal limits of network elements, the corresponding components of s_i increase. This, in turn, determines the direction of the local update of DER setpoints, encouraging correction of P_i and/or Q_i so as to return the operating state to the feasible region and restore a reliability margin [11].

Importantly, the signal s_i is node-specific. Different DERs receive different values of s_i, because their contribution to critical operating indicators – such as node voltages, loading of individual branches, or impact on active power at the feeder head – depends on their point of connection and on the electrical characteristics of the corresponding part of the network [11, 16].

The advantage of the method for RT-OPF lies in the low computational complexity of the local step, which consists of evaluating the local derivative/gradient, adjusting the setpoints in accordance with the coordination signal, and projecting onto the feasible set Y_i. Another advantage is the minimal communication requirement. In a typical implementation, each DER transmits its current measured values of active and reactive power to the aggregator, and the aggregator returns the coordination signal s_i for each DER [16]. In the basic implementation of PDGM, at each control interval, after the measurements have been received, a single update step is performed for the DER active and reactive power setpoints and, where necessary, for the coordinating dual variables, after which these setpoints are applied for the next control interval. The priority of the method is the stable correction of the operating state in closed-loop control and the maintenance of the system within the feasible region, rather than the attainment of an exact optimal solution at every interval.

The PDGM is most suitable for fast control loops (sub-second real-time operation), where low communication overhead, robustness to asynchronous updates, and scalability to a large number of DERs are critical requirements [16, 18]. The practical effectiveness of the method improves when sufficiently accurate linearised relationships are available between DER setpoints and network operating variables, and when the step-size parameters are properly tuned. This ensures stable algorithmic dynamics and minimises the risk of undesirable oscillations under rapid operating-state variations [5, 11, 18].

IV. ALTERNATING DIRECTION METHOD OF MULTIPLIERS

The ADMM method is one of the best-known approaches to solving distributed optimisation problems [17, 21, 22]. Its application is based on the combination of two key properties: first, the possibility of decomposing the original problem into local subproblems that can be solved in parallel; second, a mechanism for reconciling local solutions with global constraints by means of Lagrange multipliers and an additional quadratic penalty term [17, 21, 22]. In OPF problems, this means that local DER agents can optimise their own local objectives and account for their own technical constraints, while the aggregator ensures that these local solutions are coordinated with network constraints. Precisely because of this combination, ADMM is often regarded as a basic tool for distributed DER coordination in power-system operating-point optimisation problems [12, 17, 23].

In the standard consensus formulation of ADMM, a global consensus variable z is introduced. It consists of vectors z_i for each DER agent and represents a globally feasible version of the solution for the entire system that is compatible with the network constraints [17]. In parallel, each DER agent forms its own local solution x_i, that is, its own active and reactive power setpoints P_i and Q_i, which are acceptable from the perspective of its local objective function and technical constraints [17, 23].

The key idea of the method is to drive the system towards consensus [17], that is, towards satisfying the condition $x_i = z_i$ for each i. This means that the local solutions must agree with the corresponding components of the global solution. The aggregator determines z in such a way that it remains feasible with respect to the network constraints, while the local agents gradually adjust their x_i in the direction of this coordinated solution [17, 22]. From a geometric standpoint, ADMM may be interpreted as an alternating approximation to two sets: the set of locally feasible and locally optimal solutions of individual DERs, and the set of globally feasible network operating points. The iterative process continues until the discrepancy between these two representations of the solution becomes acceptably small [17, 22].

To ground this interaction mathematically, the global consensus optimisation problem over the distribution grid can be written as follows:

$$\min_{\{x_i\},z} \sum_{i\in I} f_i(x_i) + h(\mathrm{z}),$$
$$x_i \in Y_i, \quad x_i = z_i, \quad \forall i \in I, \tag{7}$$

where x_i and Y_i denote the local decision vector and the feasible operating set of the i-th DER, respectively, as defined in Section II; the variable $z = (z_1, z_2, \dots, z_N)$ is the global consensus variable combining the network-feasible setpoint vectors z_i of all agents; the function $h(z)$ is an indicator function representing the system-level network constraints described in Section II.B and summarised by Eqs. (2-4). $h(z) = 0$ if z satisfies the network operational limits, and $h(z) = +\infty$ otherwise.

To solve this problem distributively, the consensus formulation is mapped into the augmented Lagrangian:

$$\mathcal{L}_\rho(\{x_i\}, z, \{\lambda_i\}) = \sum_{i\in I}\Big(f_i(x_i) + \lambda_i(x_i - z_i) + $$
$$+\frac{\rho}{2}\|x_i - z_i\|^2\Big) + h(z), \tag{8}$$

where λ_i is the vector of dual variables (Lagrange multipliers) tracking the consensus discrepancy for the i-th DER, and $\rho > 0$ is a strictly positive penalty parameter controlling the strength of the consensus-enforcement term. The term $\lambda_i(x_i - z_i)$ denotes the scalar product of the dual and consensus discrepancy vectors, while $\|x_i - z_i\|^2$ represents the squared Euclidean norm that penalises the geometric distance between the local and global solutions. The coordinated solution is then obtained iteratively.

At each iteration k, after updating the local variables x_i^k and the global consensus variable z^k, the aggregator updates the dual variables according to the standard gradient ascent rule:

$$\lambda_i^k = \lambda_i^{k-1} + \rho\big(x_i^k - z_i^k\big), \quad \forall i \in I. \tag{9}$$

The dual-variable update gives the consensus mechanism its memory. If the local solution remains different from the corresponding consensus component, the multiplier λ_i^k is updated so that the next local subproblem contains a stronger incentive to reduce this discrepancy [17, 21, 22]. Thus, global coordination in ADMM is not imposed by directly assigning setpoints from a central controller. Instead, it is obtained through the repeated reconciliation of locally preferred DER setpoint vectors x_i with the corresponding components z_i of the network-feasible consensus vector. This mechanism improves the accuracy of coordination, but it also requires repeated information exchange between the local agents and the aggregator until the primal and dual residuals become sufficiently small [17, 23].

Unlike the PDGM approach, ADMM generally requires more intensive data exchange at each iteration [17, 23]. In a typical implementation, local DER agents transmit their current local solutions x_i to the aggregator, while the aggregator returns the coordinated values z_i together with service information associated with the dual variables (or equivalent coordination signals) [17]. Thus, each iteration of the algorithm involves a complete two-way communication cycle between the local agents and the aggregator [17, 23].

Since achieving an acceptable consensus accuracy often requires dozens of iterations (for example, 20-100), the aggregate communication cost increases proportionally to $O(K \cdot N)$, where K is the total number of iterations and N is the number of local agents [17, 23]. In addition, the synchronous nature of the exchange rounds makes the efficiency of ADMM sensitive to communication delays and to heterogeneity in the processing speed of individual agents, in particular to the straggler effect, where the overall speed of the algorithm is determined by the slowest participant [17]. In practice, the degree of agreement between the local and global solutions is assessed using two standard metrics [17]:

1. Primal residual, which characterises the discrepancy between x_i and z_i;

2. Dual residual, which reflects the stabilisation of changes in the consensus solution z between successive iterations.

These metrics make it possible to determine the stopping point of the iterative process in a controlled manner [17]. At the same time, in sub-second control loops there is often insufficient time to reduce both residuals to sufficiently small values, which substantially limits the practical applicability of ADMM in fast RT-OPF regimes [23].

The application of ADMM is appropriate in problems where accurate coordination between local and global solutions is critically important, and where it is possible to perform a sufficient number of iterations without stringent time constraints. Such problems include, in particular, slower control loops (seconds to minutes), planning and dispatch problems, as well as scenarios with a high-quality synchronous communication infrastructure [17, 23]. In sub-second RT-OPF control loops, where low communication overhead, a small number of iterations, and robustness to asynchronicity are decisive, ADMM often yields to methods with a simpler coordination scheme and lower data-exchange requirements [17, 23].

V. PROXIMAL ATOMIC COORDINATION METHOD

PAC belongs to the class of distributed optimisation algorithms designed for the control of large numbers of DERs under conditions of limited computational time and the need to account for the physical constraints of individual devices. Conceptually, PAC may be regarded as a development of gradient-based approaches in which the network coordination signal is augmented by a proximal mechanism that restrains excessively abrupt changes in setpoints between successive control intervals [24].

Unlike a simple gradient step, in which the new solution is determined primarily by the local derivative of the cost function and the coordinator's signal, in PAC the local update is formulated so as simultaneously to:

- reduce local undesirable effects while taking account of network requirements;
- prevent an excessively large deviation from the previous operating point;
- remain within the technically feasible region for the specific DER [24].

A key feature of PAC is that the local DER agent does not merely "move along the gradient", but instead selects a new point as the result of a small local minimisation problem [24-26]. In this problem, the following are taken into account simultaneously:

- the local cost function of the DER;
- the coordination signal from the network;
- a penalty for excessive deviation from the previous setpoint values.

In generalised form, such a step may be written as [24-26]:

$$x_i^{k+1} = argmin_{x^k \in Y_i} \Big[f_i^k(x_i) + s_i^k \cdot x^k + \\ + \frac{1}{2\alpha_i} \big|\big| x^k - x_i^k \big|\big|^2 \Big], \quad (10)$$

where x^k is the full vector of active and reactive power setpoints of all DERs at iteration k; $f_i^k(x_i)$ is the cost function (at algorithmic iteration k) that reflects the change in the local undesirable effects associated with the i-th DER when its setpoints are varied [25, 26].

From a geometric perspective, this means that the new solution is determined not only by the direction of decrease of the objective function, but also by the distance from the previous operating point. That is, the proximal term $\big|\big|x^k - x_i^k\big|\big|^2$ limits excessively large "jumps" in the setpoints by introducing a penalty for deviation from the previous operating regime [27].

In other words, the algorithm implements the principle: "move in the direction of improvement, but do not make large jumps unless necessary". It is precisely this mechanism that gives PAC its damping properties, which are particularly important in fast real-time control loops [25, 26].

The proximal term in the local problem performs several important functions. First, it limits the rate of change of the setpoints between successive iterations, even if the network coordination signal changes sharply. This reduces the risk of oscillations in the control process and improves closed-loop stability. Secondly, such a term makes it possible to account for the inertia of real equipment. In practice, inverters, batteries, and other DERs cannot always, and in some cases should not, switch instantaneously between operating modes. Accordingly, PAC is better aligned with the physical nature of controllable devices than approaches that permit abrupt setpoint changes within a single step. Thirdly, the proximal mechanism is beneficial under conditions of model error, measurement noise, and communication delay. Even if the coordination signal is inaccurate or somewhat outdated, the proximal term reduces the likelihood of an excessive response by the local DER controller [11, 24].

One of the principal advantages of PAC is that the local technical constraints of a given DER are embedded directly into the local optimisation subproblem through the set of feasible operating conditions Y_i. This is particularly important for inverter-based sources and BESS. Crucially, PAC accounts for these constraints at the stage of selecting the new solution, rather than only after a gradient step has been taken. For this reason, the control trajectory produced by PAC is often more physically consistent and more stable than in simple gradient schemes with post-facto projection [24-26].

From the standpoint of data-exchange organisation, PAC is closer to PDGM approaches than to ADMM. In a typical architecture, the aggregator forms a compact coordination signal s_i for each DER, while the local agent in return transmits its current values of P_i, Q_i and, where necessary, additional state parameters (for example, SoC for a BESS) [11, 24]. Thus, the communication volume in PAC remains relatively low since there is no need for full reconciliation of local and global variables through repeated consensus rounds, as in ADMM. However, the local computations in PAC are more complex than in the PDGM, since each DER effectively solves a small proximal minimisation problem. In practice, such a problem is often reduced to:

- a simple projection onto the feasible set;
- a small quadratic optimisation problem;
- or a closed-form analytical update for certain DER types [25, 26].

Accordingly, PAC reduces the burden on the communication infrastructure, but requires greater local computational resources on the DER side. PAC is particularly suitable in RT-OPF applications where:

- smoothness and physical consistency of control are critical;
- DERs have significant local technical constraints;
- abrupt setpoint changes are undesirable;
- the communication infrastructure does not permit the implementation of multi-iteration consensus coordination.

Compared with the PDGM, PAC generally provides better behaviour under conditions in which the technical constraints of DERs are actively limiting operation at a given moment (that is, when the system is operating close to its boundary values), as well as in the presence of errors or noise in the coordination signals. At the same time, the local implementation of PAC is generally more computationally demanding, since it involves a proximal step (local minimisation) rather than merely a gradient update followed by projection [24-26]. Compared with ADMM, PAC is less demanding in terms of communication and synchrony, although it does not provide the same degree of formal consensus coordination [24].

VI. COMPARATIVE ANALYSIS OF METHODS AND ALGORITHM SELECTION FOR RT-OPF

To identify the most effective approach for real-time DER control, a comparative analysis of three methods was carried out: PDGM, ADMM, and PAC. Since the RT-OPF problem is considered here as an iterative closed-loop process, the practical suitability of an algorithm is determined not only by its mathematical accuracy, but above all by its robustness to the infrastructural constraints of the network. For the purposes of evaluation, five key criteria were identified:

1. Number of iterations per control cycle. In a real-time algorithm, the duration of a single control interval is limited; therefore, an excessive number of internal iterations reduces performance and may render the method unsuitable for sub-second control cycles. This criterion characterises how many updates a method requires in order to achieve an acceptable correction of the operating state within a single control interval.

2. Complexity of local computations. In a distributed architecture, a significant portion of the computation is performed on the DER side (at edge controllers), which may have limited computational resources. This criterion evaluates how resource-intensive the local algorithmic step is: whether it is reduced to a simple gradient update followed by projection, or whether it requires the solution of a local subproblem (for example, a small optimisation problem) and the repeated execution of that step.

3. Communication requirements. The practical implementation of RT-OPF is constrained by the bandwidth and reliability of communication channels, so it is critically important to minimise both the volume and the frequency of data exchange. This criterion reflects whether a compact coordination signal and minimal feedback are sufficient, or whether the method requires multi-iteration two-way coordination rounds, which impair scalability when large numbers of DERs are involved.

4. Robustness to delays and asynchronicity. In real communication networks, delays, jitter, and partial loss of updates are unavoidable, and individual nodes may respond asynchronously. This criterion evaluates the tolerance of an algorithm to such conditions: whether it remains operational under partial updates, or whether it becomes highly dependent on synchronous exchange rounds and on delays at individual nodes, which may slow coordination or provoke oscillations in the operating state.

5. Constraint compliance. DER control must ensure compliance with both the local technical constraints of individual devices and network operational constraints. This criterion evaluates how local limits are achieved and how network constraints are accounted for at the aggregation level, which in turn determines the risk of operating close to boundary conditions in closed-loop control.

The results of the comparative analysis against the defined criteria are summarised in Table I, where a qualitative assessment scale (low, medium, high) is used to reflect the engineering suitability of the algorithms for RT-OPF.

To clarify how global network constraints are handled in the compared methods, it is important to distinguish their coordination mechanisms. In ADMM, global coordination is introduced explicitly through the consensus variable z, the indicator function $h(z)$ encoding the network-feasible operating region, and the dual variables λ_i, which accumulate information about the discrepancy between the locally computed DER setpoints x_i and the corresponding coordinated consensus values z_i. In contrast, PDGM and PAC do not introduce an additional consensus copy of the full setpoint vector. Instead, they rely on network-level coordination signals formed by the aggregator on the basis of measured or estimated operating conditions and the corresponding constraint sensitivities.

In PDGM, global coordination is achieved through a primal-dual tracking mechanism. The aggregator evaluates the violation or proximity of system-level constraints, such as voltage bounds, feeder-head power limits, and thermal limits, and updates the associated network-level dual variables, which act as marginal penalties for approaching or violating these constraints. These signals are then transmitted to the DER agents and included in their local projected gradient updates. Thus, although each agent updates only its own setpoint vector, the update direction is shaped by system-level information. As a result, the aggregate setpoint vector of all DER agents is iteratively driven toward an operating region that reduces the global objective while maintaining compliance with network constraints.

PAC follows the same general principle of aggregator-based global coordination, but modifies the local update by introducing a proximal regularisation term. This term penalises excessive deviations from the previous setpoint and therefore acts as a damping mechanism in the local optimisation step. Consequently, PAC tends to produce smoother setpoint trajectories than a basic projected gradient update, particularly when DERs operate close to active constraints or when coordination signals are affected by measurement noise and communication delays. In this way, PAC addresses the global RT-OPF problem through repeated locally solved proximal subproblems coordinated by network-level signals, rather than through explicit multi-iteration consensus reconciliation as in ADMM.

To illustrate these differences in a practical grid scenario, consider a sudden drop in photovoltaic generation caused by rapid cloud cover, which requires immediate grid support interventions to prevent voltage dipping. In this case, PDGM can provide a very fast response due to its single-step agility, though it may exhibit minor voltage oscillations near the boundaries. PAC can mitigate such oscillatory tendencies by penalising abrupt changes in locally computed DER setpoint vectors, thereby producing smoother trajectories toward the network-feasible operating region through its proximal damping mechanism. Conversely, the real-time control window may be insufficient for ADMM to complete the dozens of consensus iterations required for acceptable convergence, increasing the risk of delayed regulation due to communication overhead and potential latency mismatches among heterogeneous agents.

The choice of a specific method for RT-OPF implementation is not a purely mathematical issue; rather, it is an engineering compromise that depends on the degree of digital maturity of the network (including the availability of telemetry, automation, and supervisory control facilities), the computational resources available at the edge level, the quality of the communication infrastructure, and the dynamic characteristics of the controlled assets. On the basis of the analysis carried out, the following selection logic is proposed:

1. Priority on computational speed and scalability: the PDGM. This approach is most appropriate for systems with very short control intervals and large numbers of small agents, where rapid correction of the operating state and minimal data-exchange requirements are of critical importance.

Application scenario: large-scale feeders with hundreds of controllable nodes, where abrupt load changes may occur.

Selection rationale: in its basic implementation, PDGM performs a single update step at each control interval, thereby ensuring a rapid response to changes in the system state. This makes it possible to track the trajectory of a time-varying optimum without waiting for full convergence at every interval, which is important for preventing short-term violations of operational constraints. In addition, the local PDGM step is computationally simple (a gradient update followed by projection), which reduces the burden of edge computation and facilitates system scalability.

2. Balance between stability and smoothness of control: the PAC method. PAC is appropriately regarded as a proximal-gradient extension of the basic PDGM approach, providing smoother setpoint updates in networks that are prone to oscillations in the operating state.

Application scenario: systems with a high penetration of inverter-based DERs and BESS, which frequently operate near their operational limits.

Selection rationale: the proximal operator acts as a mathematical damping mechanism, reducing the risk of abrupt setpoint changes and the associated undesirable oscillations in the operating state. This contributes to smoother control dynamics and lowers the probability of undesirable transient processes in the presence of measurement noise and communication delays. An additional practical advantage of PAC is its lower dependence on multi-iteration synchronous consensus rounds than ADMM, which improves its suitability for heterogeneous communication environments. However, the edge-level computational complexity is higher than in PDGM, resulting in slower execution and imposing more stringent requirements on the edge hardware.

3. Priority on coordination accuracy in slower control loops: ADMM. The use of ADMM in sub-second RT-OPF is limited; however, it remains highly suitable for problems in which the accuracy of satisfying network constraints is more important than response speed.

4. Application scenario: secondary control loops (minute-scale cycles), optimal dispatch problems, or virtual power plant control with a reliable communication infrastructure.

5. Selection rationale: ADMM guarantees high precision in reaching a consensus between local DER interests and global network requirements. However, owing to the straggler effect and the high communication burden, its use is appropriate only when the available time budget permits dozens of data-exchange iterations between DERs and the aggregator without risking system de-synchronisation.

TABLE 1. Comparison of distributed optimisation approaches for RT-OPF.

Criterion	Primal–Dual Gradient Method	Alternating Direction Method of Multipliers	Proximal Atomic Coordination Method
Number of iterations per control cycle	Low: in the basic implementation, one iteration per control interval.	High: dozens of coordination iterations (typically 10-100) are often required for acceptable accuracy.	Low: in the basic implementation, one iteration per control interval.
Complexity of local computations	Low: a gradient step followed by projection onto the feasible set.	High: a local subproblem with a quadratic penalty term is repeated multiple times within a single control interval.	Medium: a proximal step (local minimisation/projection), more complex than PDGM, but without repeated consensus rounds as in ADMM.
Communication requirements	Low: transmission of compact coordination signals from the aggregator to each DER and actual power values in the reverse direction.	High: requires multi-iteration bidirectional data exchange at each control interval. All DERs transmit local solutions; the aggregator responds with consensus values and dual variables.	Low: transmission of compact coordination signals from the aggregator to each DER and actual power values in the reverse direction.
Robustness to delays and asynchronicity	Moderate: functional under moderate delays and partial updates, but sensitive to step-size tuning; aggressive steps may cause oscillations under noise or delay.	Low: high dependence on synchronous exchange rounds; vulnerable to delays at individual nodes.	High: proximal damping reduces sensitivity to noise and delays, lowering the risk of abrupt setpoint jumps and oscillations.
Constraint compliance	Local: via projection onto the feasible set (setpoints are sharply corrected if limits are exceeded). Network: via aggregator coordination signals (dual variables).	Local: strictly enforced (integrated into local subproblems). Network: via consensus coordination; high accuracy given a sufficient number of iterations.	Local: incorporated into the proximal step, so setpoint changes remain smooth, especially near admissible limits. Network: via aggregator coordination signals (dual variables), while the proximally damped update ensures a smoother response.

VII. CONCLUSION

This article formulates the problem of coordinated active and reactive power control for DERs as the minimisation of an aggregate cost function, subject to both local technical DER constraints and system-level network constraints. It is emphasised that, in practice, RT-OPF is implemented as a closed-loop control system with regular re-computations over time.

It is shown that real-time requirements (specifically sub-second responsiveness, low communication overheads, scalability to a large number of DERs, and robustness to latencies and asynchronous updates) are the decisive factors in selecting the algorithmic approach for aggregator-based RT-OPF systems. A comparative analysis of three distributed optimisation approaches was performed: PDGM, ADMM, and PAC.

Considering the practical constraints of the RT mode, it is substantiated that the PDGM is the most suitable base choice for RT-OPF, as it ensures minimal local computations, compact data exchange, and inherent compatibility with partial asynchronicity. It is demonstrated that PAC serves as an appropriate compromise approach when enhanced control smoothness is required; however, its local step is more complex, which limits the maximum update frequency.

It is established that ADMM ensures high precision in consensus coordination given a sufficient number of iterations; however, in a sub-second RT loop, its practical suitability is often limited by the synchronous nature of the iterations and substantial communication overheads. Therefore, it is more appropriately applied in slower loops (dispatch, planning) or in the presence of a stable synchronous communication infrastructure.

The comparison conducted is analytical in nature. Future research should be directed towards the modelling and numerical verification of algorithm performance on a typical distribution network feeder with DERs.

AUTHOR CONTRIBUTIONS

P.L. – conceptualisation, investigation and formal analysis, preparation of the draft, writing of the original; A.Z. – supervision, validation, writing-review and editing, supervision, project administration, validation, writing-review and editing.

COMPETING INTERESTS

The authors declare no conflict of interest.

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Порівняльний аналіз алгоритмів оптимального розподілу потоків потужності в мережах із розподіленою генерацією

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АНОТАЦІЯ У статті подано постановку оптимізаційної задачі та порівняльний аналіз алгоритмів оптимального розподілу потоків потужності в реальному часі (Real-Time Optimal Power Flow, RT-OPF) для розподільчих мереж із високою часткою розподілених джерел енергії (РДЕ), зокрема сонячної та вітрової генерації та акумуляторних систем накопичення енергії. Мінлива природа багатьох РДЕ та поява двонаправлених потоків потужності підвищують ризик порушення режимних обмежень, насамперед відхилень напруги та перевантажень елементів мережі, що зумовлює потребу у швидких механізмах координації РДЕ, здатних працювати у часовому масштабі часток секунди за обмеженої пропускної здатності каналів зв'язку. Задачу RT-OPF сформульовано як задачу оптимізації з обмеженнями, у якій уставки активної та реактивної потужності РДЕ вибираються для мінімізації сукупних небажаних ефектів і експлуатаційних витрат за умови дотримання локальних технічних обмежень пристроїв та мережевих режимних лімітів. Розглянуто три алгоритмічні підходи, широко описані в літературі: прямо-двоїстий градієнтний метод (Primal-Dual Gradient Method, PDGM), метод змінних напрямків множників Лагранжа (Alternating Direction Method of Multipliers, ADMM) та метод проксимальної атомарної координації (Proximal Atomic Coordination, PAC). Порівняння виконано за п'ятьма практично важливими критеріями: кількість ітерацій на один керуючий інтервал, складність локальних обчислень, комунікаційні вимоги та масштабованість, стійкість до затримок і часткової асинхронності, а також здатність забезпечувати дотримання локальних і мережевих обмежень у замкненому контурі керування. Показано, що PDGM зазвичай є найбільш придатним базовим вибором для RT-OPF у масштабних агрегаторних системах завдяки простоті локальних оновлень і компактному обміну даними, тоді як PAC забезпечує більш плавну поведінку поблизу активних обмежень ціною більш ресурсоємного локального кроку. ADMM може досягати високої точності консенсусного узгодження за достатньої кількості ітерацій, однак практично більш доцільний у повільніших контурах керування та за наявності надійної синхронної інфраструктури зв'язку.

КЛЮЧОВІ СЛОВА оптимальний розподіл потоків потужності, розподілені енергетичні ресурси, розподілена оптимізація, розподільна мережа, замкнений контур керування.