

Mathematical Modeling and Processing of Hydroacoustic Signals in Noise Environments

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ABSTRACT The article presents the results of research into methods for improving the efficiency of hydroacoustic signal transmission in complex noise environments through the use of wavelet analysis, adaptive filtering, and mathematical modeling. Hydroacoustic systems are a key tool for underwater communication, but their operation is significantly complicated by the effects of multipath propagation, water turbulence, and reflections from the surface and bottom, which lead to distortion and reduced information reliability. The work implements a structural-functional synthesis approach that combines a mathematical description of signal propagation processes, wavelet decomposition for analyzing the time-frequency structure, and adaptive filtering based on the criterion of minimizing the root mean square error. The proposed model of the hydroacoustic channel takes into account the impulse response of the medium and additive white Gaussian noise, which allows for an adequate reproduction of real transmission conditions. To improve the accuracy of information reconstruction, a combined algorithm has been developed: at the first stage, wavelet decomposition of the received signal is performed, then threshold processing of detailed coefficients is carried out according to the Donohue rule, after which Wiener or LMS-type adaptive filtering is applied. Thanks to the time-frequency localization of wavelets, effective isolation of information components is ensured even at low signal-to-noise ratios. The obtained mean square error and signal-to-noise ratio parameters demonstrate a significant improvement in reconstruction quality. The proposed approach can be used in practical hydroacoustic systems to improve their noise immunity, stability, and accuracy of information retrieval in complex underwater conditions. The combination of wavelet transformation and adaptive filtering forms the basis for the creation of intelligent signal processing systems capable of ensuring effective operation in conditions of stochastic noise and uneven propagation of acoustic waves.

KEYWORDS spectrum analysis, signal processing, acoustic signals, pulse response, scale parameter.

I. INTRODUCTION

Hydroacoustic systems are the main means of transmitting information in the underwater environment, in which electromagnetic waves suffer significant losses [1]. Modern developments in marine technology and oceanographic research are placing new demands on underwater communication systems, which must ensure high reliability of information transmission in complex hydroacoustic environments. In such systems, the main problems remain limited resolution and significant noise levels caused by multipath propagation, water turbulence, and reflections from the surface and bottom, which lead to strong temporal dispersion and random channel fluctuations [2]. This article presents a method for improving the efficiency of data transmission in hydroacoustic channels, modulated on the basis of developed mathematical models and signal processing algorithms using wavelet transformation and adaptive filtering with a Wiener filter.

II. MATHEMATICAL MODELS AND ANALYSIS METHODS

The basis of structural and functional synthesis is the construction of a model of signal transmission through a sonar channel, which is described as a linear system with a pulse characteristic $h(t)$ [3]. The input signal $s(t)$ at the output is distorted by the noise $n(t)$. As a result, the

resulting signal $r(t)$ at the output of the channel is represented in the form [4]:

$$r(t) = s(t) * h(t) + n(t), \quad (1)$$

where: $s(t)$ – input signal; $n(t)$ – noise.

Such linear convolution models are widely used in underwater acoustic channel characterization and reflect both multipath channel impulse response and Doppler-induced distortions seen in real environments [5].

To simulate a noisy environment, we will use Gaussian and color noises. Spectral density of noise power $S_n(f)$ defined as:

$$S_n(f) = N_0 \times |H_f|^2, \quad (2)$$

where: H_f – Fourier transform impulse response; N_0 – noise power.

Adaptive filtering is implemented using a Wiener filter, which minimizes the root mean square error between the signal $\widehat{s}(t)$ and the original signal $s(t)$

$$\widehat{s}(t) = \mathcal{F}^{-1} \left\{ \frac{S_s(f)}{S_s(f) + S_n(f)} R(f) \right\}, \quad (3)$$

where: $S_s(f)$ – spectral density of signal strength; $R(f)$ – Fourier transform of the received signal; \mathcal{F}^{-1} – inverse Fourier transform.

Wavelet transformation allows signals to be analyzed

locally in time and frequency [6]. Continuous wavelet signal conversion $r(t)$ defined as:

$$W_r(a, b) = \frac{1}{|a|} \int_{-\infty}^{\infty} r(t) \Psi^* \left(\frac{t-b}{a} \right) dt, \quad (4)$$

where: $W_r(a, b)$ – continuous wavelet conversion coefficients characterizing local frequency-time properties of the signal; $\Psi(t)$ – mother wavelet function, which defines the basic shape of the wavelet; Ψ^* – complex conjugation of a function $\Psi(t)$; a – a scale parameter that specifies the stretching or compression of the wavelet; $\frac{1}{|a|}$ – normalizing factor to ensure energy constancy when changing scale; b – offset parameter, which is responsible for the position of the wavelet in time.

Discrete wavelet conversion allows you to separate high-frequency noise components from low-frequency signal and effectively suppress them.

III. INTEGRATED SIGNAL PROCESSING MODEL USING WAVELET TRANSFORM

The proposed approach of structural and functional synthesis of hydroacoustic data transmission systems is based on a step-by-step combination of mathematical modeling, wavelet analysis and adaptive filtering. The key idea is to form a multi-level processing system capable of ensuring effective noise suppression and preservation of the information structure of the signal.

At the first stage, a simulation of the sonar channel is performed, which is described by the impulse response $h(t)$ and stochastic noise $n(t)$. Such a model will best reflect the real conditions of signal propagation in the underwater environment, taking into account multipath, delays and attenuation of signal energy.

The second stage is the wavelet decomposition of the received signal, which allows it to be divided into low-frequency (information) and high-frequency (noise) components. Due to the frequency-time localization of wavelets, it is possible to accurately identify areas of the signal distorted by noise, provided that useful energy is preserved.

The next step is adaptive filtering using a Wiener filter, which optimizes the signal-to-noise ratio (SNR) by minimizing the root mean square error between the estimated and actual signals. At this stage, the wavelet coefficients undergo adaptive processing in accordance with the statistical properties of the environment and the spectral characteristics of the noise.

The final step is to reconstruct the purified signal by performing an inverse wavelet transform. The result obtained is characterized by the preservation of the main structural features of the signal with a significant decrease in the noise component.

Thus, the proposed approach provides an increase in the reliability of data transmission and improvement of noise immunity in difficult noise conditions [6].

IV. MODELING AND SIGNAL PROCESSING IN A MATLAB

To confirm the effectiveness of the proposed combined approach, Numerical modeling of hydroacoustic signal transmission and processing was carried out in the MATLAB environment using standard wavelet and signal

processing toolboxes [7]. The applied time–frequency analysis and adaptive filtering techniques are consistent with recent studies on underwater acoustic channel equalization and wavelet-based denoising [8, 9]. The developed program code in MATLAB implements sequential stages of signal generation, channel modeling, noise impact simulation, wavelet-based denoising, adaptive filtering, and quality evaluation of the reconstructed signal.

First, a harmonic pulse signal with a Gaussian envelope is formed:

$$s(t) = Ae^{-\frac{(t-t_0)^2}{2\sigma_{env}^2}} \cos(2\pi f_0 t), \quad (5)$$

where: $s(t)$ – modulated signal in the time domain; f_0 – center frequency; A – signal amplitude; t_0 – temporal shift of the maximum envelope; $e^{-\frac{(t-t_0)^2}{2\sigma_{env}^2}}$ – Gaussian envelope, which determines the duration and shape of the pulse; σ_{env} – width parameter (dispersion) of the Gaussian envelope; $\cos(2\pi f_0 t)$ – harmonic filling with frequency f_0 .

Signal transmission and noise were simulated through a multipath channel model with three beams of different intensities and time delays:

$$r_{clean}(t) = \sum_{l=1}^L a_j s(t - \tau_j), \quad (6)$$

where: $r_{clean}(t)$ – cleared received signal (excluding noise); $s(t)$ – output (reference) signal; L – the number of reflected components (rays); a_j – amplitude coefficients; τ_j – the time delays of each beam, which reflect the difference between the path of the signal; $s(t - \tau_j)$ – a shifted copy of the signal corresponding to the arrival of the l -th reflected wave.

The next step is added additive white Gaussian noise with SNR = 5 level to the signal, which reproduces typical hydroacoustic signal transmission conditions in the marine environment.

In order to reduce noise, a discrete wavelet transform was applied with a basic Daubechies wavelet db4 at four levels of decomposition. The threshold processing of the corresponding coefficients was performed according to Donoho's rule [10]:

$$\lambda = \sigma_{est} \sqrt{2 \ln N}, \quad (7)$$

where: λ – the threshold value that is required to zero or reduce the corresponding wavelet conversion factors; σ_{est} – estimation of the standard deviation of noise.

$$\sigma_{est} = \frac{\text{median}(|D_1|)}{0.6745}, \quad (8)$$

where: D_1 – vector of the corresponding coefficients of the first level of wavelet decomposition; $|D_1|$ – modulus of the absolute value of the coefficients; 0.6745 – normalization coefficient, relates the median of absolute deviations to the standard deviation of the normal distribution (in our case, white Gaussian noise is superimposed, so MAD = 0.6745).

For additional improvement of the quality of the restored signal, an adaptive LMS filter with a length of $M = 32$ with an adaptation step of $\mu = 0.01$ was used.

$$w(n+1) = w(n) + \mu e(n)x(n), \quad (9)$$

where: $w(n)$ – vector of weights in the n th step; $w(n + 1)$ – updated vector of coefficients after correction; $e(n)$ – filtering error; $x(n)$ – input signal vector.

As a result of adaptation, a signal score is obtained $s(t)$, in which the useful signal will be reproduced with minimal interference (noise).

Fig. 1 shows the temporal implementations of the signal, containing:

- original signal;
- signal with noise;
- signal after wavelet filtering;
- restored signal after processing.

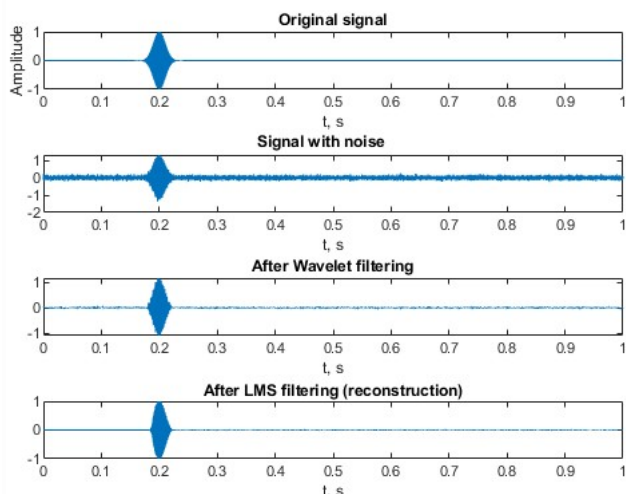


FIG. 1. Temporal implementations of the sonar signal.

Fig. 1 shows that the use of combined signal processing allows you to effectively suppress noise fluctuations while maintaining the shape of the signals.

Fig. 2 shows a comparative analysis of the signals before and after their processing. It can be seen that the fundamental harmonics are at a frequency of about 1.5 kHz, and the noise level at different bands is significantly reduced.

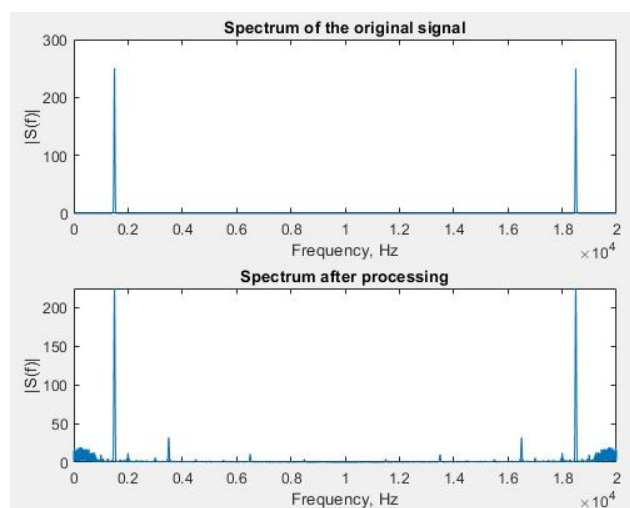


FIG. 2. Comparison of the spectra of the original signal and the signal after combined processing.

Comparative analysis of standard error (MSE) and output SNR showed a significant decrease in error and an increase in SNR value after processing. The results obtained confirm the effectiveness of the combination of

wavelet decomposition and adaptive filtering for hydroacoustic signal purification problems.

V. CONCLUSION

The proposed combined approach to the structural and functional synthesis of hydroacoustic data transmission systems, which combines mathematical modeling, wavelet analysis, and adaptive filtering, allows for the effective isolation of useful signal components and suppression of noise influences, which in turn ensures increased reliability of data transmission in complex noise environments, particularly underwater.

Modeling in MATLAB confirmed the practical effectiveness of the approach: noise cleaning using wavelet transformation and adaptive filtering significantly reduce the noise level while preserving the basic signal shape. A comparative analysis of the spectra and SNR and MSE indicators showed a significant improvement in the characteristics of the restored signal.

The results demonstrate that the proposed method can be successfully applied to real hydroacoustic data transmission systems, increasing their noise immunity and accuracy of information reproduction.

AUTHOR CONTRIBUTIONS

Y.P. – writing (original draft preparation), conceptualization, methodology, investigation; H.L. – writing (original draft preparation), conceptualization, methodology, investigation.

COMPETING INTERESTS

The authors declare no competing interests.

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Математичне моделювання та обробка гідроакустичних сигналів у шумових середовищах

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ABSTRACT У статті наведено результати дослідження методів підвищення ефективності передавання гідроакустичних сигналів у складних шумових середовищах шляхом застосування вейвлет-аналізу, адаптивної фільтрації та їх математичного моделювання. Гідроакустичні системи є ключовим інструментом забезпечення підводного зв'язку, проте їх функціонування істотно ускладнюється впливом багатопроменевого поширення, турбулентності водної середовища, відбитків від поверхні та дна, що призводить до спотворень та зниження достовірності інформації. У роботі реалізовано підхід структурно-функціонального синтезу, який поєднує математичний опис процесів поширення сигналів, вейвлет-декомпозицію для аналізу часово-частотної структури та адаптивну фільтрацію на основі критерію мінімізації середньоквадратичної похибки. Запропонована модель гідроакустичного каналу враховує імпульсну характеристику середовища та адитивний білий гаусовий шум, що дозволяє адекватно відтворити реальні умови передачі. Для підвищення точності реконструкції інформації розроблено комбінований алгоритм: на першому етапі здійснюється вейвлет-декомпозиція прийнятого сигналу, далі проводиться порогова обробка детальних коефіцієнтів за правилом Доного, після чого застосовується адаптивна фільтрація Вінера або LMS-типу. Завдяки часово-частотній локалізації вейвлетів забезпечується ефективне виділення інформаційних компонент навіть за низького співвідношення сигнал/шум. Моделювання в середовищі MATLAB підтвердило результативність комбінованої обробки: порівняння часових та спектральних характеристик сигналів до і після фільтрації засвідчило зменшення шумових компонентів та збереження основної структури. Отримані показники середньоквадратичної похибки та співвідношення сигнал/шум демонструють суттєве покращення якості реконструкції. Запропонований підхід може бути використаний у практичних гідроакустичних системах для підвищення їх завадостійкості, стабільності та точності відновлення інформаційних повідомлень у складних підводних умовах. Поєднання вейвлет-перетворення та адаптивної фільтрації формує основу для створення інтелектуальних систем обробки сигналів, здатних забезпечити ефективне функціонування в умовах стохастичних завад та нерівномірного поширення акустичних хвиль.

KEYWORDS аналіз спектрів, оброблення сигналів, акустичні сигнали, імпульсна характеристика, масштабний параметр.



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