

BRIDGING THE GAP BETWEEN KACHINSKY AND FAO/USDA PARTICLE-SIZE DISTRIBUTION SYSTEMS: MATHEMATICAL MODELING AND PEDOGENETIC HARMONIZATION

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This study addresses the methodological challenges of converting soil particle-size distribution (PSD) data from the regional Kachinsky system to international FAO/USDA standards, focusing on the diagnostic of lithogenic heterogeneity in Retisols.

We compared various parametric models (Fredlund 4P, van Genuchten, Skaggs) and non-parametric spline functions. A specific Log-Linear Sectional Interpolation (LLSI) algorithm was developed to estimate the 2 μm clay threshold. To bridge the 1–2 mm data gap inherent in the Kachinsky method, a fractal power-law scaling approach was applied for sand fraction extrapolation.

Traditional fixed-ratio conversions (e.g., physical clay/2) proved inaccurate, yielding classification errors of 30–50%. The proposed LLSI algorithm, combined with fractal scaling and pedogenetic corrections for organic matter and carbonates, achieved high predictive accuracy ($R^2 > 0.95$). Furthermore, we quantified the systematic bias of laser diffraction (LD), which underestimates clay content by 8–15% compared to sedimentation methods due to particle non-sphericity.

Effective harmonization requires continuous modeling of the PSD curve rather than simple arithmetic coefficients. The integration of the LLSI method and fractal extrapolation provides a robust framework for incorporating regional soil archives into global databases, ensuring the accurate representation of complex soil textures like those of the Precarpatic Retisols.

Keywords: Soil texture, Particle-size distribution (PSD), Kachinsky system, FAO/WRB standards, LLSI algorithm, Fractal scaling, Retisols, Lithogenic heterogeneity

Introduction. The disparity in methodologies for determining soil texture (particle-size distribution) remains one of the most formidable challenges in modern soil science. This inconsistency hinders the development of unified digital soil maps and the integration of regional databases into global frameworks such as SoilGrids or the World Reference Base for Soil Resources (WRB). In Ukraine, as well as across much of Eastern Europe and Central Asia, the foundational system remains Kachinsky's classification, developed in the mid-20th century (Laktionova, 2011). Conversely, the international scientific community, aligned with FAO and USDA standards, employs a different fractional classification. A critical divergence lies in the definition of the clay fraction: the international threshold is 2 μm , whereas Kachinsky's system defines "physical clay" as particles $<10 \mu\text{m}$ (Bhatt et al., 2025).

The challenge of data conversion extends beyond simple mathematical recalculation; it

involves fundamental differences in physicochemical soil pretreatment, dispersion techniques, and measurement principles—specifically, sedimentation-based methods (governed by Stokes' Law) versus Laser Diffraction (LD) (Shein, 2009).

Kachinsky's system is based on a binary principle: the division of soil into "physical sand" (particles $>0.01 \text{ mm}$) and "physical clay" (particles $<0.01 \text{ mm}$) (Laktionova, 2011). This threshold was originally chosen to reflect a distinct shift in agrophysical properties; once the physical clay content exceeds 10%, the soil exhibits cohesion, plasticity, and structural development (Callesen et al., 2023). In contrast, the international system (FAO/WRB), which mirrors the USDA approach, utilizes a tripartite model (sand, silt, and clay) with boundaries at 2, 50, and 2000 μm (Moreno-Maroto & Alonso-Azcárate, 2022).

These taxonomic differences lead to a scenario where identical textural class names (e.g., "medium

loam") correspond to significantly different quantitative particle ratios across systems. Consequently, direct comparison is precluded

without the application of pedotransfer functions or mathematical transformations (Table 1).

Table 1.

Correlation between FAO/USDA and Kachinsky particle-size distribution systems based on fraction diameters

Fraction Name (FAO/USDA)	Particle Diameter (μm)	Corresponding Kachinsky Fractions (μm)
Clay	< 2	Partial "clay" (< 1) and "fine silt" (1–5)
Silt	2 – 50	Sum of fractions: 1–5, 5–10, and 10–50 ("medium silt" and "coarse silt")
Sand	50 – 2000	Sum of fractions: 50–250, 250–1000, and gravel > 1000

One of the most critical discrepancies lies in the upper size limit defined for particle-size analysis. In the Kachinsky method, analysis is traditionally performed on fine earth with a diameter of <1 mm. Conversely, international standards define the <2 mm fraction as the fundamental baseline for determining soil texture. This fundamental difference necessitates the use of extrapolation models or pedotransfer functions to estimate the

sand content within the 1–2 mm range, which is otherwise excluded in the traditional Soviet-era protocol (Shangguan, W., et al, 2014).

Table 2 presents the particle-size ranges employed across various international systems and national schools of soil science. These data highlight the complexity of data transfer caused by the lack of alignment between fractional boundaries.

Table 2.

Particle size distribution boundaries (μm) according to various international systems and national soil science standards

Fraction Name	Ukraine	FAO/WRB	ISSS	USA (USDA)	USA (ASTM/Other)	Canada	England & Wales	Germany (DIN)	Poland (PTG)	France	Australia
Colloids	<0.1	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2
Clay	<1	<2	<2	<2	<5	<2	<2	<2	<2	<2	<2
Silt	1–50	2–20	2–20	2–50	5–50	2–50	2–60	2–60	2–50	2–50	2–20
<i>Fine silt</i>	1–5	-	-	-	-	-	2–6	-	-	-	-
<i>Medium silt</i>	5–10	-	-	-	-	-	6–20	-	-	-	-
<i>Coarse silt</i>	10–50	-	-	-	-	-	20–60	-	-	-	-
Sand	50–1000	63–2000	20–2000	50–2000	50–2000	50–2000	60–2000	60–2000	50–2000	50–2000	20–2000
<i>Very fine sand</i>	-	63–125	-	50–100	-	50–100	-	-	50–100	-	-
<i>Fine sand</i>	50–250	125–200	20–200	100–250	50–250	100–250	60–200	60–200	100–250	-	-
<i>Medium sand</i>	250–500	200–630	-	250–500	-	250–500	200–500	200–500	250–500	-	-
<i>Coarse sand</i>	500–1000	630–1250	200–2000	500–1000	250–2000	500–1000	500–2000	500–2000	500–1000	-	-
<i>Very coarse sand</i>	-	1250–2000	-	1000–2000	-	1000–2000	-	-	1000–2000	-	-
Gravel	>1000	>2000	>2000	>2000	>2000	>2000	>2000	>2000	>2000	>2000	>2000

Mathematical Methods for PSD

Interpolation and Approximation. To facilitate the transition between different classification systems, it is necessary to reconstruct a continuous particle-size distribution (PSD) function. Two primary approaches are most widely utilized: statistical regression between fixed fraction contents and the approximation of the cumulative PSD curve using parametric models (Shangguan et al., 2014).

Parametric Distribution Models A broad spectrum of functions is employed to describe soil PSD, ranging from simple log-linear equations to complex four-parameter models. Research indicates that the performance and efficiency of a specific model are highly dependent on the soil textural class (Shangguan et al., 2014).

The Fredlund Model is described by the following equation (Fredlund et al., 2000):

$$F(d) = \frac{1}{\{\ln[\exp(1)+(a/d)^n]\}^m} \left\{ 1 - \left[\frac{\ln(1+d_f/d)}{\ln(1+d_f/d_m)} \right]^7 \right\}$$

where d is the particle diameter; a , n , and m are curve shape parameters; d_f and d_m are constants defining the boundary conditions for sedimentation.

Utilizing this model enables the calculation of the mass percentage for any specified diameter, particularly the 2 μm and 50 μm thresholds, which are essential for conversion to the FAO system (Sadovski, Ivanova, 2020).

Table 3.
Parametric models for PSD approximation and their optimal application ranges

Model Name	Number of Parameters	Optimal Application and Performance
Fredlund – F4P (Fredlund et al., 2000)	4	Universal application; high precision for heavy-textured (clayey) soils
van Genuchten – VG (Haverkamp & Parlange, 1986)	2	Hydrophysical modeling; optimal for medium-textured soils
Weibull – W (Assouline et al., 1998)	3	Light sandy soils and coarse-grained deposits
Anderson – AD (Andersson, 1990)	4	Frequently demonstrates the best goodness-of-fit (R^2) for loams
Skaggs – S (Skaggs et al., 2001)	2	Simple approximation when dealing with limited input data

Spline Interpolation. A current trend in soil science is the use of non-parametric methods, such as log-cubic splines and non-uniform rational B-splines (NURBS) (Marhoul et al., 2025). These methods ensure the cumulative PSD curve passes exactly through the measured data points, preventing the unrealistic oscillations common in high-degree polynomials. Log-cubic spline interpolation demonstrates a Root Mean Square Error (RMSE) of approximately 6.3%, outperforming traditional log-linear models (Shang, 2013).

Regional Harmonization Algorithms. One of the most robust approaches for converting Kachinsky-based data is the regression algorithm proposed by Shein (2009). This method is founded on extensive comparative datasets across various soil genetic types. It was established that the direct conversion of "physical clay" ($<10 \mu\text{m}$) to USDA

"clay" ($<2 \mu\text{m}$) must account for the soil-forming environment (pedogenesis). For instance, Chernozems and forest soils require specific coefficients that reflect the mineralogical maturity of their fractions.

The algorithm utilizes regression equations of the form $y = ax + b$, where y represents the FAO fraction content and x denotes the corresponding Kachinsky fractions. Studies confirm that accurate conversion requires data from at least six Kachinsky fractions to construct a reliable PSD curve, from which values for 2, 50, and 2000 μm are derived.

The Bulgarian school of soil science introduced a unique approach using discrete mathematics and set theory (Venn diagrams) to analyze the intersections of intervals between classification schemes (Sadovski, Ivanova, 2020). Rousseva (1997) found that exponential functions better

describe heavy-textured soils, while power-law functions are more suitable for light sandy soils.

For practical implementation in Bulgaria (which utilizes an adapted Kachinsky system), the following protocol is applied:

1. Fraction intervals $A = [a_1, b_1]$ and $B = [a_2, b_2]$ are defined.

2. Their intersection is calculated as $AB = [\max(a_1, a_2), \min(b_1, b_2)]$.

3. A weighting coefficient for each fraction is calculated based on the integral distribution function.

In our research concerning the diagnostics of lithogenic heterogeneity in Retisols (Pecarpatican region, Ukraine) based on particle-size distribution (Nikorych, 2015), we applied a specialized conversion algorithm (*The Log-Linear Sectional Interpolation (LLSI) method*) as follows:

Step 1. Carbonate Correction. Losses from HCl treatment during sample pretreatment for sedimentation analysis are distributed proportionally across all particle-size fractions.

Step 2. Data Series Preparation. We construct a cumulative PSD curve using the following log-transformed coordinates:

log(d), mm	-3	-2.3	-2.0	-1.3	-0.6	0
Fractional Content	C*	C+FS	C+FS+MS	C+FS+MS+CS	C+FS+MS+CS+fS	Cumulative Sum

* C (Clay <1 μm); FS (Fine Silt 1–5 μm); MS (Medium Silt 5–10 μm); CS (Coarse Silt 10–50 μm); fS (Fine Sand 50–250 μm).

Step 3. Determination of the 2 μm Threshold. We identify the unknown value corresponding to $x = -2.7$ (the 2 μm boundary), which lies on the cumulative curve between points $x_1 = -3$ (1 μm) and $x_2 = -2.3$ (5 μm).

Although the overall integral curve often follows a sigmoid function, such as $y = a \cdot \text{arctg}(bx)$ (Fig. 1), the segment between these two proximal points is treated as a straight line.

Thus, the unknown point (x, y) (Fig. 2) is calculated using the linear interpolation equation:

$$y = \frac{x - x_1}{x_2 - x_1} \cdot (y_2 - y_1) + y_1$$

where:

- y = clay content (<2 μm result);
- $x = -2.7$; $x_1 = -3$; $x_2 = -2.3$;
- y_1 = Kachinsky C fraction (<1 μm);
- y_2 = Sum of Kachinsky C+FS fractions (<5 μm).

We derive the final calculation formula:

$$y = \frac{-2.7 - (-3.0)}{-2.3 - (-3.0)} \cdot (y_2 - y_1) + y_1$$

After simplifying the coefficients:

$$y = 0.43 \cdot (y_2 - y_1) + y_1$$

Given that in the Kachinsky system, $(y_2 - y_1)$ represents the content of the fine silt fraction (FS, 1–5 μm and y_1 is the clay fraction (C<1 μm), the simplified conversion formula is:

$$y = 0.43 \cdot FS + C$$

Step 4. Silt Determination. After calculating y (clay content at 2 μm), we fix the value at $x = -1.3$ (50 μm boundary) as the cumulative sum. The silt content is then determined by subtracting the calculated clay content from this sum.

Step 5. Sand Determination. The total sand content is calculated by the difference: 100 - (clay + silt).

Step 6. Classification. The final textural class is determined using the USDA textural triangle (also known as the Feret triangle). This algorithm is easily automated via MS Excel.

Since the Kachinsky method typically terminates at the 1 mm threshold, there is a data deficit regarding the full sand fraction as defined by FAO/USDA (2 mm threshold). To resolve this, we utilize the concept of fractal self-similarity in soil mass (Rousseva, 1997). Assuming that the particle distribution in the coarse fraction follows a power-law scaling, the cumulative fraction for 2 mm ($F(2)$) is estimated based on the values for 1 mm and 0.5 mm:

$$F(2) = 10^{2 \log(F(1)) - \log(F(0.5))}$$

This approach ensures the harmonization of Kachinsky data with international databases. Without this correction, the omission of the 1–2 mm fraction may lead to an erroneous classification of the soil as being finer-textured than it actually is (Shangguan et al., 2014).

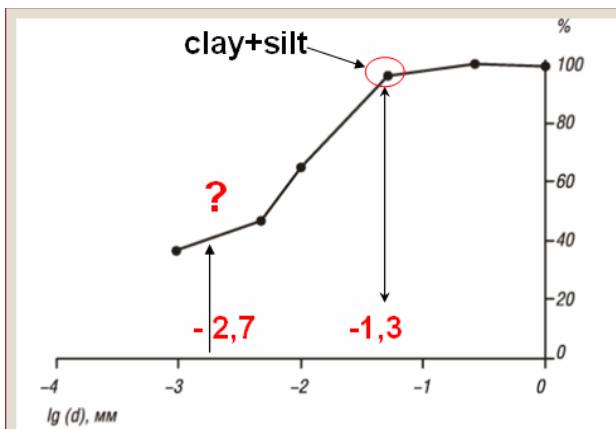


Fig. 1. Cumulative PSD curve and input data points

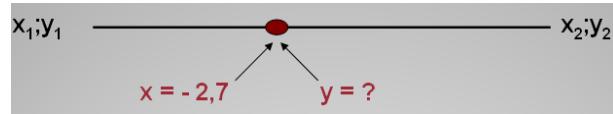


Fig. 2. Segment of the cumulative curve indicating the coordinates of the target point (corresponding to the clay fraction)

New Methods – New Methodological Challenges.

The transition to modern Laser Diffraction (LD) has introduced a new layer of complexity to the conversion problem. Although LD is significantly faster and requires a smaller sample mass, its physical principles differ fundamentally from the sedimentation analysis (e.g., pipette or hydrometer methods) upon which both the Kachinsky and FAO systems are based.

Laser diffraction measures the volume distribution of particles based on light scattering patterns, whereas the pipette method measures mass distribution based on hydrodynamic resistance and settling velocity (Stokes' Law) (tabl. 4). Because soil particles, particularly clay minerals, are non-spherical (platy or needle-like), LD often overestimates the size of these particles, as they scatter light as if they were spheres with a larger equivalent diameter (Yang et al., 2015).

Table 4.
Comparison between Sedimentation and Laser Diffraction (LD) Methods

Characteristic	Sedimentation (Kachinsky/Pipette)	Laser Diffraction (LD)
Physical Principle	Stokes' Law (viscous drag)	Mie or Fraunhofer theory (optics)
Distribution Type	Mass-based (%)	Volume-based (%)
Particle Shape	Affects velocity (ellipsoids settle slower)	Affects scattering (particle orientation)
Clay Determination	Typically yields a higher % of $<2 \mu\text{m}$	Often underestimates clay ($<2 \mu\text{m}$) by 8–15%

To compensate for these discrepancies, logistic transformation functions should be utilized to convert LD data into "pipette-equivalent" values (Callesen, 2023). It has been established that using raw LD data without correction leads to soil classification errors in 43% of cases for the Kachinsky system and 65% for the USDA system.

The Role of Sample Pretreatment and Organic Matter Removal.

Methodological divergences begin at the pretreatment stage. The "classical" Kachinsky protocol often lacks mandatory removal of organic matter (OM) and carbonates, relying instead on chemical dispersants (NaOH , $\text{Na}_4\text{P}_2\text{O}_7$) and boiling.

In contrast, international standards (ISO 11277, USDA) mandate the prior oxidation of OM using hydrogen peroxide (H_2O_2). Organic matter acts as a cementing agent, forming stable microaggregates that the Kachinsky method may misidentify as larger particles (e.g., silt or fine sand). This leads to an artificial "lightening" of the perceived texture in humus-rich soils like Chernozems. Removing OM significantly increases the recorded clay content, making the pretreatment stage arguably more critical for conversion accuracy than the mathematical model itself.

Software and Automation in PSD Conversion.

Modern pedometrics utilizes specialized digital tools to manage these complex calculations, primarily within the R programming environment:

- Soiltexture package: Enables the construction of textural triangles (USDA, FAO, Kachinsky) and facilitates automated conversion via log-linear interpolation.
- Plotrix and ggtern packages: Used to visualize sample distributions in ternary plots, highlighting the shift in textural classes when changing classification schemes.
- NURBS in MATLAB: Employed for high-precision approximation of PSD curves when source data are non-uniform or contain significant gaps.

Conclusions. The meta-analysis of literature indexed in Scopus and Web of Science confirms that while converting particle-size results between the Kachinsky system and FAO/WRB standards is scientifically feasible, it demands a transition from simplistic arithmetic to rigorous mathematical and pedological modeling.

Direct recalculation via fixed ratios, such as the common "physical clay/2" approximation, is demonstrably inadequate for high-impact research

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due to unacceptable error margins of 30–50%. Instead, the priority must be shifted toward constructing continuous cumulative PSD curves using robust parametric models like Fredlund 4P or monotonic cubic splines, which ensure mathematical consistency across classification thresholds.

Furthermore, the accuracy of such conversions is inextricably linked to soil genesis and pretreatment protocols; specifically, the failure to remove organic matter in historical Kachinsky-based datasets requires the application of pedogenetic correction coefficients to avoid the artificial "lightening" of soil textures.

As the field moves toward laser diffraction (LD), researchers must remain vigilant regarding its tendency to underestimate clay content and implement system-specific calibrations that account for particle non-sphericity. Ultimately, achieving global data interoperability requires a commitment to transparency, where researchers report not only the qualitative textural class but also the quantitative values for the 2, 50, and 2000 μm fractions derived from continuous mathematical approximations.

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ГАРМОНІЗАЦІЯ СИСТЕМ ГРАНУЛОМЕТРИЧНОГО СКЛАДУ КАЧИНСЬКОГО ТА FAO/USDA: МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ТА ПЕДОГЕНЕТИЧНИЙ АСПЕКТ

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У статті розглянуто методологічні аспекти конвертації даних гранулометричного складу ґрунтів із регіональної системи М. О. Качинського у міжнародні стандарти FAO/USDA. Основну увагу приділено діагностиці літогенної неоднорідності ретисолів (*Retisols*). Автором проведено порівняльний аналіз параметричних моделей (*Fredlund 4P*, *van Genuchten*, *Skaggs*) та непараметричних сплайн-функцій. Для визначення вмісту фракцій мулу (< 2 мкм) розроблено алгоритм лог-лінійної секційної інтерполяції (*LLSI*). Для компенсації відсутності даних у діапазоні 1–2 мм за методом Качинського застосовано підхід фрактального степеневого масштабування для екстраполяції піщаних фракцій.

Встановлено, що традиційні методи перерахунку за фіксованими коефіцієнтами (зокрема, «фізична глина / 2») призводять до помилок класифікації у 30–50% випадків. Доведено, що застосування алгоритму *LLSI* у поєднанні з фрактальним масштабуванням та врахуванням вмісту органічної речовини й карбонатів забезпечує високу точність прогнозування ($R^2 > 0.95$). Виявлено систематичну похибку методу лазерної дифракції, який недооцінює вміст мулу на 8–15% порівняно з седиментаційними методами. Обґрунтовано, що ефективна гармонізація даних потребує безперервного моделювання інтегральної кривої розподілу часток, що дозволяє коректно інтегрувати регіональні ґрунтові архіви до глобальних баз даних.

Ключові слова: текстура ґрунту, гранулометричний склад, система Качинського, стандарти FAO/WRB, алгоритм *LLSI*, фрактальне масштабування, бурувато-підзолисті ґрунти, літогенна неоднорідність.

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